Stabilizer Statistical Mechanics

W E Salazar[1], G Saxena[2], J S Baker[2], T H Kyaw[2], and L C Kwek[1,3]

[1]*Centre for Quantum Technologies, Singapore, Singapore*
[2]*LG Electronics Toronto AI Lab, Toronto, Canada*
[3]*National Institute of Education, Nanyang Technological University, Singapore, Singapore*
Contact Email: william_esteban@u.nus.edu

Magic is a necessary resource for universal quantum computation, yet quantifying the magic of quantum states remains a computationally hard problem. Recently, stabilizer Rényi entropy was proposed as a computable magic monotone based on the Pauli spectrum. However, these quantities only capture the even moments of the Pauli spectrum, providing only partial information about the underlying statistical distribution. To recover this missing information, we introduce a new framework, stabilizer statistical mechanics, whose central object is the stabilizer partition function, which simultaneously encodes all moments arising from the Pauli spectrum. We further use this function to characterize different classes of quantum states throughout Hilbert space. Our construction reveals a clear and previously undetected distinction for pseudo-magic states that evaded earlier formulations. In addition, we develop an efficiently computable magic measure based on this framework and prove that it is a faithful magic monotone.