Light Propagation in a Turbulent Atmosphere as Diffusion in the Space of Spatial Modes

K Kravtsov[1], S Amairi-Pyka[1], and J A Grieve[1]

[1] *Quantum communications, TII, Abu Dhabi, United Arab Emirates*
Contact Email: konstantin.kravtsov@tii.ae

Most Free-Space Optical (FSO) communication channels are affected by turbulent fluctuations in the refraction index in the air. They lead to beam spreading and loss of spatial coherence. Although, there are established numerical methods for modeling these channel imperfections, they do not provide sufficient insight into the underlying processes.

In this work, we approach the problem analytically and study behavior of an orthogonal mode set such as Hermite- or Laguerre-Gaussian propagating in the turbulent medium. Turbulence manifests itself in the optical phase randomization that follows a certain spatial frequency spectrum. These phase fluctuations break the orthogonality of the chosen mode set and effectively couple different modes together. We show that the coupling strength is proportional to the frequency-weighted power of phase fluctuations, i.e. the variance of the frequency-filtered fluctuations, where the filter is defined by the pair of the interacting modes. Moreover, this implies that the coupling strength is proportional to the channel length should the mode size and the turbulence conditions remain uniform throughout the channel. Thus, the propagation process is effectively a linear diffusion of optical power from a single spatial mode at the input of the channel to other modes in the same orthonormal set.

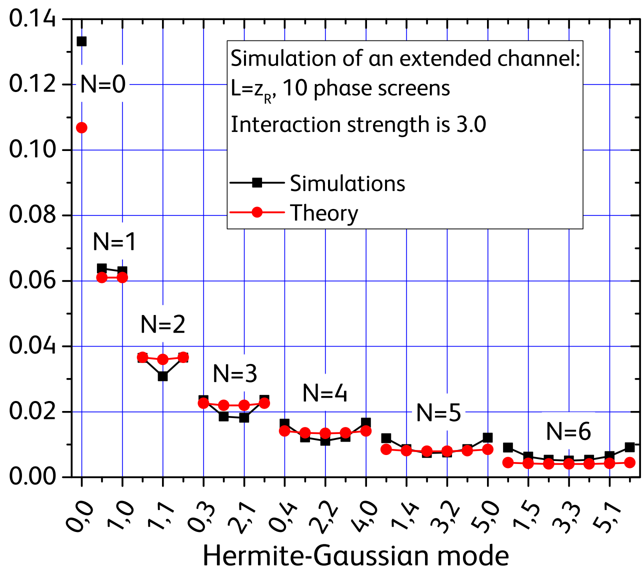

Figure 1: Comparison between the developed theory and full-scale simulations for a channel with strong turbulence, corresponding to the Rytov parameter of $\sigma_R^2 = 3.2$. Plotted is the distribution of optical power among 28 lowest-order Hermite-Gaussian modes for the Gaussian beam (the fundamental mode) passing through a turbulent atmospheric channel. The diffusion matrix $\mathbf{\Lambda}$ was truncated to those 28 modes. Modes are presented in the following order: (0,0), (0,1), (1,0), (0,2), (1,1), (2,0), *etc.*

Effectively, the system is described by a symmetric real-valued matrix $\mathbf{\Lambda}$ of coupling rates Λ_{ab} between modes a and b. For a uniform channel of length L and mode a at the input, the vector $\mathbf{v}$ of the average output power (or probability) distribution among modes is given by the matrix exponential $\mathbf{v} = \exp(\mathbf{\Lambda} L)\mathbf{e_a}$, where $\mathbf{e_a}$ is a vector with the 1 at a-th position and zeros elsewhere. The matrix $\mathbf{\Lambda}$ has negative values on the main diagonal and positive ones elsewhere. Moreover, the sum of all elements in each row (column) is zero, so the power is conserved, while the stationary solution to which the system converges at $L \to \infty$ is a uniform distribution of power among all available modes. A minor detail is that this infinite matrix has to be truncated to some finite size for practical purposes. The truncated matrix exhibits some loss of total power due to diffusion to unaccounted higher-order modes.

In Fig. 1 we demonstrate the output of our model for strong turbulence. For weaker turbulence levels our model gives excellent agreement with simulations, which proves validity of our analytic approach.