Wave Packet Dynamics in Quantized Electromagnetic Fields: Analytic Insights into Electron–Light Interaction

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We present a fully analytic treatment of the dynamics of a free electron interacting with a multimode quantized electromagnetic field. Starting from a nonrelativistic Hamiltonian in the dipole approximation, we derive a closed-form solution for the coupled electron–field system by applying a momentum-dependent displacement transformation, which diagonalizes the Hamiltonian in a composite momentum–photon-number basis.

This formalism allows us to describe the time evolution of arbitrary electron wave packets interacting with general quantum states of light, including coherent, Fock, and squeezed states. Explicit expressions are obtained for the expectation value and uncertainty of the electron position, enabling a direct comparison with classical dynamics. We find that the expectation value closely follows the classical trajectory, with only small corrections arising from the quantized nature of the field. In contrast, the spatial uncertainty of the electron exhibits a pronounced dependence on the quantum state of the radiation field.

In particular, we show that quantum fluctuations of the electromagnetic field are directly mapped onto the spreading of the electron wave packet. While coherent states reproduce classical-like behavior, nonclassical states such as squeezed and Fock states lead to qualitatively different dynamics. For squeezed radiation, the electron wave-packet variance can be periodically reduced below the coherent-state case, depending on the squeezing parameters and phase. In multimode scenarios, relevant for realistic laser pulses, this effect is governed by the spectral composition and phase coherence of the field, leading to constraints on the bandwidth for observing fluctuation suppression.

Our results provide a transparent analytical framework for understanding how quantum properties of light influence electron motion in strong-field processes. This approach offers intuitive insight into recent experiments on quantum-optical effects in high-field physics and establishes a basis for exploring the role of nonclassical radiation in ultrafast and attosecond phenomena.