Solvable Model of Induced Interactions: Finite Bose Systems and the Infinite-Particle-Number Limit

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We consider a multiple-species mixture of interacting bosons, N_1 bosons of mass m_1, N_2 bosons of mass m_2, and N_3 bosons of mass m_3 in a harmonic trap of frequency ω. The corresponding intraspecies interaction strengths are λ_{11}, λ_{22}, and λ_{33}, and the interspecies interaction strengths are λ_{12}, λ_{13}, and λ_{23}. When the shape of all interactions are harmonic, this is the generic multiple-species harmonic-interaction model which is exactly solvable. We start by solving the many-particle Hamiltonian and concisely discussing the ground-state wavefunction and energy in explicit forms as functions of all parameters, the masses, numbers of particles, and the intraspecies and interspecies interaction strengths. We then move to compute explicitly the reduced one-particle density matrices for all the species and diagonalize them, thus generalizing the treatments in [1-3]. As an application, we investigate two species with no interspecies interaction between them, and interacting with a third species which serves as a bath. The bath thus induces interaction between the two species. Some exact results on the induced interactions and correlations between the two species, for finite Bose systems and at the infinite-particle-number limit, are presented and discussed. Applications for bosons in cavities are briefly mentioned [4].

References

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