Harnessing Superresolution: Continuous Wavefunction Dynamics

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In recent years, there has been a significant focus on the development of technologies that leverage quantum optical effects. A critical aspect of this development is the ability to predict the evolution of quantum optical states across various processes. However, numerical simulation of the dynamics of quantum systems presents substantial challenges due to the high dimensionality of Hilbert space.

To simulate an evolution of quantum states, one has to solve the time-dependent Schrödinger equation. A widely adopted approach implies a search for a solution in the Fock-state representation. However, the size of the Hilbert space is proportional to the amount of photons present in the system. This observation imposes a challenge for systems with high photon numbers. The alternative way is to consider continuous variables (CV), *i.e.* position or momentum basis. In numerical simulation, a continuous variable is discretized to the extent necessary to resolve all the details of the encoded wave function.

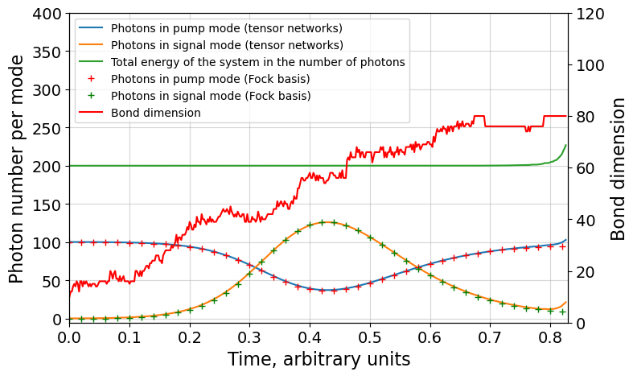

Figure 1: Photon number dynamics of pump and signal modes (left Y-axis) and bond dimension (right Y-axis) during quantum evolution in the process of spontaneous parametric down-conversion (SPDC): pump mode is initialized in the coherent state $|\alpha\rangle = |10\rangle$. Blue and orange lines present photon number dynamics using tensor networks; crossed dots demonstrate simulation in the Fock basis; green line is the total energy expressed in the number of photons; red solid line represents dynamics of MPS bond dimension

It turns out that describing wave functions with exponentially high precision is possible with the help of tensor networks. The Matrix Product State (MPS) is the simplest one-dimensional tensor network, offering scalability and numerical tractability. In this work, we use MPS for the efficient encoding of both wave functions and quantum operators. MPS representation provides exponential compression for both, wave-function and a Hamiltonian of interest. The time-dependent wave function is propagated using direct numerical solution of Schrödinger equation in MPS format.

To validate our approach, we simulate the process of degenerate spontaneous parametric down-conversion (SPDC) in time. First, we consider the dynamics of signal plus pump wave-fuction where the pump mode is initialized in the coherent state $|\alpha\rangle = |10\rangle$. This regime allows a direct computation in the Fock basis, serving as a benchmark for verification. A comparison of the time-evolved density matrices obtained via Fock-basis calculations and our tensor network framework confirms exact agreement (see figure 1), demonstrating the accuracy of our method.

We further extended our analysis to a more computationally demanding regime, initializing the pump mode in a higher-amplitude coherent state $|\alpha\rangle = |100\rangle$. In this case, Fock-basis simulations become intractable due to quadratic growth of the Hilbert space size with the amplitude of the coherent state. However, our tensor network approach remains computationally feasible, since MPS encoding of smooth functions is efficient.

Thus, our tensor network method enables superresolution of the wave function across the entire time range of quantum evolution. Also, theoretical and empirical considerations suggest that this approach is capable of simulating the quantum dynamics of several interacting optical modes. This can be especially important to simulate the dynamics of optical modes in systems as whispering gallery mode microresonators.