Zernike Polynomials on Quantum Phase Space: Aberrations of Generation Probabilities of Entangled Photon Pairs

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The quantum states of light have long been playing an important role in the physics of lasers and parametric processes in quantum optics, e.g. in noise reduction in measurements. In quantum imaging or quantum information science entangled photon pairs are applied, to produce non-classical, *e.g.* Einstein-Podolsky-Rosen (EPR)-type correlations. The subject of the talk belongs to this quite broad area of research, and the theoretical results to be presented may hopefully be useful for further studies.

We shall present and discuss the exact photon statistics of EPR photon pairs stemming from the non-degenerate down-conversion process, which we have derived partly on the basis of our earlier works [1,2]. It will be shown that in a special case the derived probability amplitudes (matrix elements of the evolution operator between number states) reduce to the Zernike polynomials. The Zernike polynomials are well-kown in the classical theory of aberration in optical imaging [3], and also have important applications (e.g. in the controll of deformable mirrors or in wavelet analysis [4]). In the physical system discussed here the 'aberration' stems from the (optional) induced emission and absorption, so it quantifies the deviation from the spontaneous process. The derived quantum Zernike polynomials form a complete set of orthogonal aberration functions of the phase-space variables (parameters of the generation) of the photons. We shall illustrate how the quantum aberration, as we propose to call this phenomenon, may manifest itself in the statistics of the photon-pair emission. Though these functions do not depend directly on the coordinates in real space where the electromagnetic wave propagates, they likely have an effect on the wavefront. Thus, it seems that the quantum Zernike moments may also be used for quality-assessment of parametric sources of entangled photon pairs. Besides, the derived transition matrix elements can also be interpreted as overlap integrals in real space mode functions. Such an analogy has already been used quite recently, on the basis of our general formula for the squeezing matrix elements [2], which have been considered as overlap integrals between Hermite-Gauss modes [5] of an optical cavity.

References

[1] S Varró, Phys. Scr. **90**, 074053 (2015)

[2] S Varró, New J. Phys. **24**, 053035 (2022)

[3] M Born and E Wolf, Principles of Optics, Cambridge University Press, Cambridge, 2002

[4] M Pap, in: P W Hawkes (ed.), Advances in Imaging and Electron Physics, Vol. 188, Academic Press, San Diego, 2015, p. 79

[5] W J Hughes, T H Doherty, J A Blackmore, P Horak and J F Goodwin, arXiv:2306.05929v1 (2023)