Optimizing States for Quantum-Enhanced Interferometry: Two Case Studies

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Quantum-enhanced interferometry allows one to go beyond the classical shot-noise limit (SNL), thus allowing a more efficient use of the input resource, quantified via the average number of input photon, $\bar{N}$ [1]. The optimal phase sensitivity performance is often expressed via the quantum Cramér-Rao bound (QCRB), $\Delta\varphi_{QCRB} = 1/\sqrt{\mathcal{F}}$ where $\mathcal{F}$ denotes the quantum Fisher information (QFI) [2,3]. One can distinguish between the single- and two-parameter QFI, relevant when an external phase reference is, and, respectively, is not available [4,5].

When the discussion is limited to pure and non-entangled input states, Lang & Caves have shown that, if one input is a coherent source ($|\alpha\rangle = \hat{D}(\alpha)|0\rangle$ where the displacement operator is $D(\alpha) = e^{\alpha\hat{a}^\dagger - \alpha^*\hat{a}}$), the two-parameter QFI is maximized when the second input port is fed by a squeezed vacuum state [6]. The same authors showed in reference [7] that if one assumes a pure and non entangled input, among the states featuring a variable (*i.e.* non-fixed) number of photons, the optimal input state is a double squeezed vacuum.

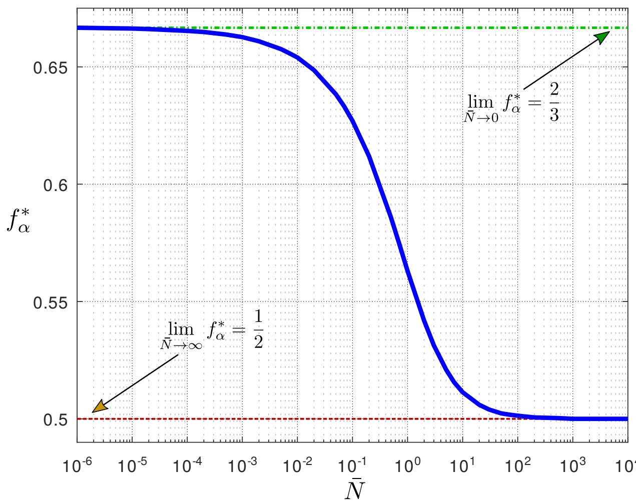

Figure 1: The optimum coherent power ratio f_α^* (thick blue curve) that maximizes the two-parameter QFI versus the average input photon number $\bar{N}$ for a coherent plus squeezed vacuum input state

In this work we address both previously mentioned scenarios and answer the following question: what power ratio between one input state and the total average number of photons $\bar{N}$ optimizes the phase sensitivity? Contrary to previous studies, we allow the Mach-Zehnder interferometer (MZI) to be unbalanced and also address both detection scenarios *i.e.* when having access – or not – to an external phase reference [8].

For the case of a coherent plus squeezed vacuum input we show that an optimal coherent ratio $f_\alpha = |\alpha|^2/\bar{N}$ that maximizes the two-parameter QFI can always be found in closed form (see Fig. 1). We also show that we have the limits $f_\alpha = 2/3$ ($f_\alpha = 1/2$) when $\bar{N} \to 0$ ($\bar{N} \to \infty$). For the double squeezed vacuum we reproduce and extend the results from previous references. We also show that for this input state and a detection scheme having access to an external phase reference, the optimal measurement scheme is the one employed in reference [9].

References

[1] C Caves, Phys. Rev. D **23**, 1693 (1981)

[2] C Helstrom, Phys. Lett. A **25**, 101 (1967)

[3] S Braunstein and C Caves, Phys. Rev. Lett. **72**, 3439 (1994)

[4] M Jarzyna and R Demkowicz-Dobrzanski, Phys. Rev. A **85**, 011801(R) (2012)

[5] S Ataman, Phys. Rev A **102**, 013704 (2020)

[6] M Lang and C Caves, Phys. Rev. Lett. **111**, 173601 (2013)

[7] M Lang and C Caves, Phys. Rev. A **90**, 025802 (2014)

[8] K Mishra and S Ataman, Phys. Rev. A **106**, 023716 (2022)

[9] J A H Nielsen, J S Neergaard-Nielsen, T Gehring and U L Andersen, Phys. Rev. Lett. **130**, 123603 (2023)