Two-Mode Squeezing and Entanglement in Atomic Boson Sampling

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Recently, an atomic boson sampling of excited atom occupations in an equilibrium gas with a Bose-Einstein condensate (BEC) has been suggested as a process that could be $\sharp$P-hard for classical computing [1]. An example of a multi-qubit BEC trap [2] shows that it could serve as a fruitful platform for studying various phenomena associated with atomic boson sampling and quantum supremacy. It is a platform alternative to and drastically different from the photonic boson sampling in a linear interferometer actively discussed in the literature for more than a decade.

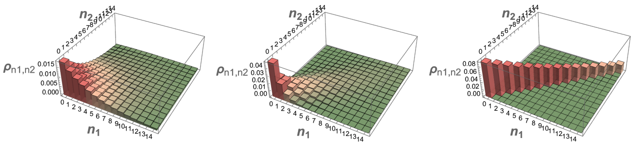

Figure 1: The joint probability distribution ρ_{n_1,n_2} for atomic boson sampling from two excited bare-atom modes formed by a unitary mixing of two different eigen-squeeze modes – the sine and cosine standing waves with the same wave vector $\mathbf{k}$. Here the absolute value of the eigen-squeeze-mode anomalous correlator $\alpha = |\langle \hat{a}_{\mathbf{k}} \hat{a}_{\mathbf{k}} \rangle|$ is increasing from the value equal to the slightly less than eigen-squeeze-mode normal correlator $\eta = \langle \hat{a}^{\dagger}_{\mathbf{k}} \hat{a}_{\mathbf{k}} \rangle = 10, \alpha = 8$, (left graph), to the value $\alpha = 10$, (center graph), and then to its maximum value $\alpha = \sqrt{\eta(1+\eta)}$, (right graph), leading to increasingly pronounced squeezing and, hence, entanglement of the sampled bare-atom modes

This talk is based on our recent analysis [3] of the simplest possible model of the BEC trap – the box with the periodic boundary conditions, in which the condensate is uniform. This commonly accepted, textbook model allows us to greatly simplify the general theory outlined in [1,2] and explicitly disclose the manifestations of and the mechanism behind the $\sharp$P-hard computational complexity of atomic boson sampling. Using the hafnian master theorem [4] and Bloch-Messiah reduction of the Bogoliubov transformation we analytically calculate the sampling probability distribution via a matrix hafnian. We show that its computing is $\sharp$P-hard since the squeezing and entanglement of excited-atom modes are simultaneously and naturally present in the interacting BEC gas due to an interplay with the eigen-squeeze modes and eigen-energy quasiparticles.

In this talk we focus on the effects of two-mode squeezing and entanglement. We illustrate the properties, manifestations and mechanism of appearance of $\sharp$P-hardness by a series of examples suggested by [3], such as the one shown in Fig. 1. They provide a clear vision of expected observations helpful for designing proof-of-principle experiments on the realization of atomic boson sampling systems. Compared to the remarkable experiment on fluctuations of the total noncondensate occupation [5], the experiments on atomic boson sampling do not imply counting all noncondensed atoms. Nontrivial patterns of the joint probability distribution of atom numbers for two counter-propagating plane waves as in Fig. 1 could be obtained in the experiments similar to the experiment on the full counting statistics [6].

References

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