Regular Phase Coherent States and the Möbius Transformation of the Exponential Phase Operator of the Photon

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Recently we have introduced the regular phase coherent states [1] as eigenestates of a new exponential phase operator of photon modes, whose eigenvalues belong to the open unit disc of the complex z-plane. These special SU(1,1) coherent states can be generated from vacuum e.g. by an interaction with intensity-dependent coupling, or in a parametric down-conversion process, where the complex number z depends on the parameters of the source. The evolution of the state is described by the unitary operator U(g), where the group element g also contains the time parameter. In the present contribution first we show that, by projecting the states of the Hilbert space of the photon mode to the regular phase coherent states, the received funcionals are analytic functions of z on the open unit disc, and they form the so-called weighted Bergman space [2]. The effect of unitary representation operators on this space (apart from a multiplier) results in Möbius transformations (fractional-linear transformations) of the argument of the analytic functions. By using the normal ordering of the exponentials of the SU(1,1) generators [3], we have also calculated the matrix elements of the evolution operator U(g) between number eigenstates. In a special case these matrix elements reduce to Zernike polynomials, which have long been used in wavefront analysis in classical optics, and they also appear in hyperbolic wavelet analysis [4,5]. Finally we show that the unitary transformation of the exponential phase operator, induced by U(g), yields the Möbius (or Blaschke) function of this operator, leading to a new aspect of the quantum phase problem.

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