Universal Nature of Quantum Supremacy

V V Kocharovsky[1]

[1] *Department of Physics and Astronomy, Texas A&M University, Mitchell Physics Building, 4242 TAMU, 578 University Dr., College Station, TX, USA. Contact Phone: +1-979-3244923*
Contact Email: vkochar@physics.tamu.edu

We disclose the universal nature of computational ♯P-hardness and quantum supremacy of quantum many-body systems [1,2]. This problem is one of the central problems in modern quantum physics. We consider a generic example of many-body interacting systems – a trapped BEC-gas of interacting Bose atoms, apply a newly found hafnian master theorem [3] and implement a famous Toda's theorem on a ♯P-complete oracle.

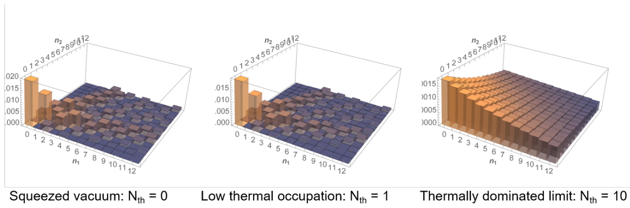

Figure 1: Joint occupation probabilities for two interfering squeezed excited-atom states: Nontrivial quantum statistics, which manifests ♯P-hard complexity of many-body systems, is hiding under thermal fluctuations as thermal occupation N_{th} increases

Atomic boson sampling of noncondensed atom numbers in an interacting BEC-gas is a new platform for studying quantum supremacy [1,2,4]. It is very different from photonic boson sampling in a linear interferometer widely studied in the last decade. We present a detailed analytical theory of atomic boson sampling for a BEC-gas in a box trap [1], in particular, the sampling probability distribution and its characteristic function. We find that two necessary ingredients of the ♯P-hardness, squeezing and interference, are naturally present in the BEC gas even in equilibrium. The existence of squeezing in the interacting BEC gas has been known since [5]. Contrary to Gaussian boson sampling of noninteracting photons in a linear interferometer, atomic boson sampling does not require usage of external sophisticated sources of bosons in quantum squeezed states.

We suggest proof-of-principle experiments designed to demonstrate manifestations of the ♯P-hard complexity of atomic boson sampling [1,2,4]. Extracting joint probability distribution for the occupations of just two excited atom states (say, two counter-propagating plane waves or their unitary mixed counterparts as in Fig. 1) would be already a remarkable experimental achievement [1]. Such experiments are even easier for implementation than the ones on statistics of the total noncondensate occupation [6].

We conclude that the nature of the quantum supremacy and ♯P-hard complexity has a universal origin – an intuitively obvious complexity of computing the multivariate integral in Fourier-series coefficients of a sign-indefinite strongly-oscillating function. It is related to ♯P-hardness of computing a matrix hafnian (or permanent) [1,3,7] which behaves like a lacunary or fractal function with exponentially wide spectrum.

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