From the Figure-8 Motion to Nonlinear Thomson and Compton Scattering

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We discuss the exact relativistic description of both the classical and the quantum mechanical motion of charged particles interacting with a laser field, which is represented by an electromagnetic plane wave of arbitrary intensity. This subject gives us also a good opportunity to remind us to Compton's discovery [1], which has been an important ingredient of quantum electrodynamics (QED). First, in a brief historical overview we shall highlight the question of wave-particle duality and simultaneity in Compton scattering [2,3]. The exact solutions of the Lorentz and Dirac equations of a charged particle in a plane electromagnetic wave in vacuum are represented by the well-known figure-8 motion and the Volkov states, respectively. These solutions have been extensively used in describing high-harmonic emission and attosecond pulse generation in the Compton or Thomson scattering processes [4]. In this part of the talk we shall analyse the surprising mathematical connection between the relativistic figure-8 motion in a monochromatic radiation and the classical Kepler-Coulomb trajectories in a central field [5].

By going beyond the external field approximation, the Dirac (Klein-Gordon or Schrödinger) equation of the joint system of a charged particle interacting with quantized radiation modes in vacuum can also be solved exactly in various cases [6,7]. The photon part of the single-mode stationary states are squeezed (coherent) number states, whose photon statistics has been determined quite recently in terms of Gegenbauer polynomials [8], and this is now can be used for a refined analysis of high-harmonic generation in a quantized radiation field, which was first studied by us long ago [6].

Finally we discuss the interaction of a charged Dirac particle with two co-propagating circularly polarized quantized modes. On the basis of the recently found new exact solutions, we shall see that entangled photon pairs naturally appear in this system, like in the quantum optical non-degenerate parametric down-conversion process. In a special case the probability amplitudes of the distribution of these photon pairs reduce to Zernike functions, which are well-kown in the theory of aberration in classical optical imaging. Accordingly, it is justified to introduce the concepts of aberration and Zernike moments on quantum phase space, which may give a new aspect in the theoretical description of non-perturbative QED interactions and quantum optical parametric processes, as well.

References

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