The Imaginary Part of the High-Order Harmonic Cutoff

E Pisanty[1,2,3], M F Ciappina[3,4], and M Lewenstein[3,5]

[1] Theory Department, Max Born Institute for Nonlinear Optics and Short Pulse Spectroscopy, Berlin, Germany
[2] Department of Physics, King's College London, London, UK
[3] Quantum Optics Theory, ICFO - The Institute of Photonic Sciences, Barcelona, Spain
[4] Guangdong Technion - Israel Institute of Technology, Shantou, China
[5] ICREA-Institució Catalana de Recerca i Estudis Avançats, Barcelona, Spain
Contact Email: emilio.pisanty@kcl.ac.uk

We present a rigorous definition of the high-harmonic cutoff which is applicable to arbitrary driver waveforms. Our definition provides a natural meaning for the imaginary part of the cutoff energy, which controls the strength of quantum-path interference in the plateau. This construction radically simplifies quantum-orbit calculations. Using this definition, we build the Harmonic-Cutoff Approximation to calculate the exact location and brightness of the cutoff at a fraction of the cost of the state of the art – giving access to a much wider class of optimization tasks.

The harmonic cutoff is one of the key concepts in high-harmonic generation, but it is also an elusive object which has long escaped a precise definition. In this work we provide the first natural definition for the harmonic cutoff; we show how to find it, in a simple and computationally effective way; and we show how it can be used.

We build our construction on a new type of quantum orbit, which recollides at the harmonic-cutoff times t_{hc} defined by a *second*-order saddle-point equation for the usual action S,

$$\frac{\mathrm{d}^2 S}{\mathrm{d}t^2}(t_{\mathsf{hc}}) = 0. \tag{1}$$

The cutoff energy is then given by the real part of $\Omega_{\mathsf{hc}} = \frac{\mathrm{d}S}{dt}(t_{\mathsf{hc}})$. This provides a natural value for $\mathrm{Im}(\Omega_{\mathsf{hc}})$, which controls the strength of quantum-path interference in the plateau. The quantum orbits emerge as different branches of a unified Riemann surface, which are united at branch points – the harmonic-cutoff times t_{hc}.

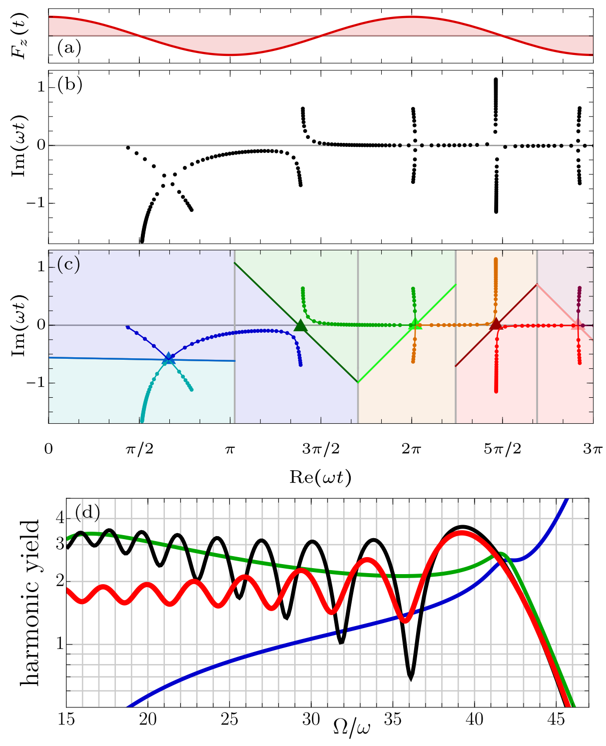

Figure 1: The saddle-point equations for HHG produce an unstructured cloud of solutions, shown in (b), which then need to be classified into quantum orbits. The harmonic-cutoff times t_{hc} (triangles, in (c)) simplify this classification. (d) The HCA (red line) matches the saddle-point approximation (black line, combining the short (blue) and long (green) trajectories) at the cutoff

More practically, the times t_{hc} can be used to classify the solutions of saddle-point equations into quantum orbits in a simple and efficient way, making calculations more flexible and effective.

Last (but not least), the information at the times t_{hc} is enough to fully characterize the harmonic spectrum at the cutoff. This allows us to build a new approach – the Harmonic-Cutoff Approximation (HCA) – which gives a quantitatively accurate estimate of the location and brightness of the cutoff, and a qualitative estimate of the spectrum in the upper plateau. This requires solving a *single* saddle-point equation – as opposed to dozens or hundreds of equations, as in previous approaches.

References

[1] E Pisanty, M F Ciappina and M Lewenstein, J. Phys. Photonics **2**, 034013 (2020)