The Hafnian Master Theorem, Wick Theorem and Gaussian Integrals: Atomic Boson Sampling in a BEC Trap Versus Gaussian Boson Sampling in an Optical Interferometer

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Recently we calculated anomalous quantum fluctuations in the total number of noncondensed atoms in an interacting Bose-Einstein-condensed gas confined in a finite-size trap [1]. Simultaneously, the first experimental observation of such BEC number fluctuations has been fulfilled [2]. Despite being non-Gaussian and highly nontrivial, the statistics of the total noncondensate occupation can be calculated analytically by means of the Wick theorem and Gaussian integrals and computed by classical computers in polynomial time.

Now it is time to start the analysis of even deeper problems of many-body quantum statistical physics of BEC – the ♯P-hard problems which require exponential time for classical computing. Recently we proved [3] that computing the joint probability distribution of excited-state occupations in the BEC trap is such a problem. Our proof is based on the newly discovered Hafnian Master Theorem [4], which allows one to express the joint occupation probabilities via a hafnian of a matrix associated with the normally-ordered covariance matrix. A similar, classical MacMahon Master Theorem providing an explicit formula for the generating function for matrix permanents had been known since 1916. However, a formula for the generating function for hafnians had been missing until now. Both permanents and hafnians are ♯P-hard for computing and constitute a universal tool for studying ♯P-hard problems in quantum computing, computer science, physics of many-body systems *etc.* [5].

The above problem of joint occupations statistics is essentially the problem of atomic Boson Sampling analogous to the well-known photonic Boson Sampling in a linear interferometer [6,7]. Both atomic and photonic platforms are directly related to the concept of quantum supremacy. In essence, an experiment on atomic Boson Sampling can be viewed as a more detailed experiment on atom number fluctuations in BEC when, instead of counting the total number of noncondensed atoms, one counts separately the numbers of atoms in different fractions of the noncondensate associated with the prescribed subgroups of the excited states. The physics of N atoms in a BEC trap looks substantially different from the physics of massless photons in the interaction-free, nonequilibrium (nonthermal), linear interferometer due to the presence of the condensate, thermal equilibrium, particle mass and interaction as well as the absence of external sources of bosons. We show these peculiarities do not prevent solving this problem analytically and, in fact, turn the BEC trap into a promising platform for testing quantum many-body physics, *e.g.*, Boson-Sampling quantum supremacy.

This talk is planned as an introduction to a cluster of the fascinating problems stated above, including the definition and meaning of the matrix permanent and hafnian, as well as the differences between them. In particular, the Hafnian Master Theorem as a universally powerful tool in the many-body quantum statistical physics will be explained and compared against the well-known Wick theorem, which is the basis of the modern many-body statistical physics and quantum field theory. A simple proof of the Hafnian Master Theorem by means of the Gaussian integrals will be outlined as well. Also, an intuitively clear derivation of the Hafnian Master Theorem's formula via its profound quantum-statistical meaning and Wigner transform will be sketched.

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