Photonic Circuits for Performing Minimum Quantum State Tomography of Path Qudits

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The use of integrated optical circuits opens new possibilities for the generation, manipulation, and characterization of high dimensional states besides the ease of transmission of these states through an optical fiber. We propose photonic circuits to perform minimum quantum state tomography of path qudits and show how to determine all the constituents parameters of these circuits (directional couplers and phase shifters) [1]. Our strategies were based on the symmetries of the involved POVMs (positive operator-valued measures) suggested for minimum tomography and allowed us to obtain interferometers smaller than those obtained by other already known methods. The calculations of the transmittances and reflectivities of the beam splitters were made using the definition of probability operators in extended Hilbert spaces and the application of Naimark's theorem. The employment of equidistant states for the definition of the POVM elements allowed us to develop a recipe applicable to the tomography of qudits of any dimension, generalizing our scheme. All necessary POVM elements are implemented at the same time by the static circuit and measured at the different photonic circuit exits simultaneously. All necessary measurement probabilities are obtained from the photon counts at the circuit outputs in one measurement time interval. This is important because it minimizes the noise introduced by the experimental apparatus during the detection time. The proposed tomography method uses SIC-POVM (symmetric informationally complete positive operator-valued measure).

Three-dimensional structures fabricated in glass or crystals are studied in various research groups currently. We also propose a 3D photonic circuit designed to perform full quantum state tomography in one-qudit systems of any finite dimension $N \geq 3$ represented in the form of photon path states for a single photon in one of the waveguides printed in the glass [2]. The qudit input-state is simply a superposition on the basis formed by these states. Our strategy involves dividing the interferometer into three distinct unitary operations that act independently between layers or vertical sectors. One consequence is that we have achieved a considerable reduction in the complexity of the photonic circuit. In our proposal, the number of beam splitters scales better than in other designs over generic transformations. The same happens with the optical depth, quantity defined as the maximum number of beam splitters traversed by a photon from its input port until its output port from the circuit.

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References

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