Complementarity Beyond Wave-Particle Duality: A Historic Perspective

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Einstein, in 1905, in his explanation of the photoelectric effect, postulated that light, the quintessential wave, had to possess particle-like properties. In the course of 1923-24, de Broglie, analyzing electron scattering from metal surfaces, postulated that electrons, the quintessential particles, must possess wave-like properties. In 1928, Bohr made the first attempt to reconcile the two viewpoints and introduced the concept of complementarity (or, in a more restricted sense, wave-particle duality), and thus the now 90 years history of complementarity has started.

We begin with a brief overview of the history of quantitative complementarity relations. A particle going through an interferometer can exhibit wave-like or particle-like properties. The first quantitative duality relation was obtained by Greenberger, and Yasin [1], between the strictly single-partite properties: predictability $P = |\rho_{11} - \rho_{22}|$ and visibility $V = 2|\rho_{12}|$ and has the form

$$P^2 + V^2 \leq 1. \tag{1}$$

In a seminal study of the two-path interferometer, Englert introduced detectors into the interferometer arms and defined the path distinguishability, D, as the discrimination probability of the path detector states [2]. He derived a relation between this type of path information and the visibility $V = 2|\rho_{12}|$ of the interference pattern, in the form

$$D^2 + V^2 \leq 1. \tag{2}$$

In a follow-up [3], Englert and Bergou showed that D is a joint property of the system and the meter to be clearly distinguished from predictability, which is a strictly single partite property. They showed that (2) corresponds to the so-called which-way sorting (post-selection) of the measurement data. They also introduced the quantum erasure sorting, which led to the duality relation $P^2 + \mathcal{C}^2 \leq 1$, where the coherence $\mathcal{C}$ is a joint property of the system and detectors. Most importantly, they conjectured that D should be related to an entanglement measure. Taking up this conjecture, the complete bipartite (particle-meter) complementarity relation, connecting complementarity, i.e., visibility of the interference pattern, V, and path predictability, P, to entanglement, was found in [4], in the form of a *triality relation*,

$$P^2 + C^2 + V^2 \leq 1. \tag{3}$$

Here C is the concurrence, emerging naturally as part of the completeness relation for a bipartite system. In [5], this *triality relation* was further generalized to multi-path (n-path) interferometers. These works completed the research on quantitative complementarity and brought the Bohr-Einstein debate to a very satisfying closure. In particular, Eq. (3), which is a triality relation, displays explicitly that entanglement is the genuinely quantum contribution with no classical counterpart, whereas visibility, quantifying wave-like behavior, and predictability, quantifying particle-like behavior, can be regarded as a classical contribution.

In all of the works discussed above, the l_2 measure of coherence was employed. Recently, however, a resource theory of quantum coherence was developed, and two new coherence measures were introduced [6]. The l_1 measure is the trace distance, the entropic measure is the entropic distance of a given state to the nearest incoherent state. In the second part of the talk, we present our recent results for multi-path interferometers, employing the new measures. Using these measures, we derived entropic and l_1

based duality relations for multi-path interferometers [7, 8]. The l_1 based duality relation for n-path interferometers is

$$\left(\frac{C+D-\frac{n-2}{n-1}}{\frac{\sqrt{n}}{n-1}}\right)^2+\left(\frac{C-D}{\sqrt{\frac{n}{n-1}}}\right)^2\leq 1,\quad C,D>0, \tag{4}$$

where C is the l_1 measure of coherence, generalizing the visibility V. To close, we will present recent results generalizing duality relations to finite groups [9] and discuss recent entropic duality relations [10].

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