Nonlinear Waves in a Dispersive Vacuum

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We adopt [1] an electro-magnetic Lagrangian that involves higher-order derivatives of the wave vector potential, and that is constructed so as to include the dependence of the invariant photon mass on the so-called photon quantum nonlinearity parameter χ_γ. In the case of an electromagnetic configuration that depends on a single spatial coordinate, say x, and where the fields correspond to a single polarization, this Lagrangian can be written as

$$\mathcal{L} = -\frac{m_e^4}{4\pi\alpha}\left[\frac{B^2-E^2}{2} - \epsilon_2\,\frac{(B^2-E^2)^2}{4} + \epsilon_3\,\frac{(B^2-E^2)^3}{8}\right] \tag{1}$$
$$-\frac{\mu\, m_e^4}{4\pi\alpha}\left\{B^2\left[\partial_x\left(E^2-B^2\right)\right]^2 + E^2\left[\partial_t\left(E^2-B^2\right)\right]^2 - 2EB\left[\partial_x\left(E^2-B^2\right)\partial_t\left(E^2-B^2\right)\right]\right\},$$

where $\epsilon_2 = 2\alpha/(45\pi)$, $\epsilon_3 = 32\alpha/(315\pi)$ and $\mu = 4\alpha/(135\pi)$. This Lagrangian allows us to describe dispersive effects in the interaction of two counter-propagating light pulses by a nonlocal extension of the nonlinear wave equation that is derived from the Heisenberg-Euler Lagrangian.

In the case of a finite-amplitude wave impinging on a large amplitude counter-propagating low-frequency wave (the so-called cross fields configuration), we show that this higher-order derivative Lagrangian leads to Korteveg-de Vries type soliton solutions. We derive the soliton width and propagation velocity in terms of the soliton amplitude and the cross fields.

References

[1] F Pegoraro and SV Bulanov, Phys. Rev. D **103**, 096012 (2021) and references therein