Phase Transitions in Quantum Complex Networks

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At present complex networks evoke increasing interest among the scientific community performing advanced studies at the boundary of information science, statistical physics, quantum optics and communication [1]. In particular, these studies aim to investigate complex processes in social networks and on the Internet, in living systems integrated within biological networks, designing new (quantum) materials with complex topology and structure, developing communications and information network technologies, including quantum networks.

We establish a phase transition problem occurring in complex networks, which may be recognized as quantum communication network systems [2]. We suggest the Dicke-Ising model, which generalizes the Dicke and Ising models for annealed complex networks presuming spin-spin interaction. The model concerns the spin-1/2 (two-level) systems located in the network nodes and placed in the local classical (magnetic) and weak quantum transverse (photonic) fields. The regular, random, and scale-free networks are examined. To describe paramagnetic (PM)-ferromagrenic (FM) and superradiant (SR) phase transitions, we introduce two order parameters: the total weighted spin z component and the normalized transverse field amplitude, which correspond to the spontaneous magnetization in z and x directions, respectively. For the regular networks and vanishing external field, we demonstrate that these phase transitions generally represent prerequisites for the crossover from a disordered spin state to the ordered one inherent to the FM and/or SR phase. Due to the interplay between the spin interaction and the finite-size effects in networks, we elucidate novel features of the SR state in the presence of the PM-FM phase transition. For the scale-free networks, we demonstrate that the network architecture characterized by the particular value of degree exponent, which plays a key role in the SR phase transition problem. Within the anomalous regime, complex networks possess a strong, effective spin-spin interaction supporting a fully ordered FM state, which is practically non-sensitive to variations of the quantum transverse field or moderate classical magnetic field. In the scale-free domain, the ordering in the network spin system appears as a result of the interplay between magnetizations in x and z directions. We establish the conditions for the network parameters, classical and quantum field features to obtain a quantum phase transition in the network system when the critical temperature approaches zero. Our work opens new perspectives for quantum Internet design.

References

[1] S Dorogovtsev, A Goltsev and J F Mendes, Rev. Mod. Phys. **80**, 1275 (2008)

[2] A Yu Bazhenov, D V Tsarev and A P Alodjants, Phys. Rev. E, in press