The probability of high harmonic generation (HHG) as well as the polarization properties of generated harmonics are determined by the laser-induced dipole moment $\vec{D}(\Omega)$. In the low frequency limit this dipole moment may be efficiently evaluated in the framework of quantum orbits and $\vec{D}(\Omega)$ may be presented as a sum of partial dipole moments associated with closed classical electron trajectories, which start (ionization time) and end (recombination time) at complex moments of time [1]. In the low frequency limit, the imaginary part of the ionization time is much less than the characteristic period of an intense laser field. The smallness of the ratio of these two times allows us to develop a formal perturbation theory for the calculation of ionization and recombination times for an HHG process [2] and calculate analytically the HHG amplitude for a laser pulse of arbitrary shape.

Within the proposed approach, the dipole moment $\vec{D}(\Omega)$ can be also presented as a coherent sum of partial dipole moments. However, in contrast to the quantum orbits approach, they are associated with closed classical trajectories, which start and end at some real times. For instance, in the first order of perturbation theory, the liberated electron starts to move along a closed trajectory at those times that ensure a minimum kinetic energy at the moment of ionization, while returning (recombination) times are given by the moments for which the kinetic energy $E$ of the electron in the laser field coincides with the recombination energy of the electron in the continuum $E = \Omega - I_p$, where $\Omega$ is the harmonic energy and $I_p$ is the ionization potential.

The HHG yield for the simplest case of a bound s-state can be presented in the factorized form:

$$Y(\Omega) \propto \Omega^4 |\vec{D}(\Omega)|^2 = W(E)\sigma_{\text{rec}}(E),$$  \hspace{1cm} (1)

where $W(E)$ is the electronic wave packet (EWP) and $\sigma_{\text{rec}}(E)$ is the recombination cross section. The analytic expression for the EWP can be presented in the form:

$$W(E) = \Omega \sqrt{2E} \left| \sum A_{\text{det}} A_{\text{prop}} \right|^2,$$  \hspace{1cm} (2)

where $A_{\text{det}}$ is the detachment amplitude in the adiabatic approximation [3] for a given electron momentum $\vec{p} = - \int_{t_i}^{t_f} \vec{A}(t)dt/(t_f - t_i)$, and $t_i$ and $t_f$ are real ionization and recombination times. The propagation amplitude $A_{\text{prop}}$ is expressed in terms of the classical action and its magnitude is of the order of $(t_f - t_i)^{-3/2}$.

In order to extend our model results to the case of neutral atoms, we replace the detachment amplitude by the Coulomb-corrected ionization amplitude [4] and recombination cross section by its corresponding atomic counterpart. We check the accuracy of our extension by comparing our analytical results with results of numerical solution of the 3D time-dependent Schrödinger equation (TDSE) for a bicircular field with carrier frequencies $\omega$ and $2\omega$ and the same intensity for both components. We performed such a comparison for a wide range of frequencies $\omega$ corresponding to $1.6 \mu m \leq \lambda \leq 3 \mu m$ for fixed intensity $I = 10^{14} \text{ W/cm}^2$. Our comparisons show excellent agreement between the TDSE and analytic results for harmonics with $\Omega > u_p$, where $u_p = I/4\omega^2$. In our numerical analysis we also discuss the dependence of the HHG yield in bicircular fields on the time delay between the two circularly polarized components of the bicircular laser field.

Acknowledgements: This work was supported by the Russian Science Foundation Grant No. 15-12-10033 (MVF, AAS, NVV), by NSF Grant No. PHYS-1505492 (AFS), and by the Ministry of Education and Science of the Russian Federation through Grant No. 3.1659.2017 (AAM).
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