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Natural Intensity and Intermode Beating Frequency Fluctuations in Two-Mode He–Ne and He–Ne/CH_4 Lasers

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Abstract – The influence of the degree of intermode coupling and mode position on the spectrum of natural intensity fluctuations and intermode beating frequency for a two-mode laser is theoretically and experimentally studied. A new physical phenomenon – non-linear resonances in the spectral density of natural fluctuations of intermode beating frequency and natural intensity fluctuations in a two-mode He–Ne / CH_4 laser is observed. Dependence of resonance profiles obtained on laser operating parameters is also investigated.

1. INTRODUCTION

Currently gas lasers are widely used in development of laser measuring systems [1 - 5]. In this field two-mode lasers appear to be highly promising. Such lasers are applied to perform measurements of lengths and displacements [6, 8, 9], refractive indices of samples [7], and optical anisotropy [10, 11]; they are also used in accelerometers [10], quantum frequency standards [14, 15], highly sensitive spectroscopes [16, 17], etc.

Both laser radiation intensity and intermode beating frequency can be viewed as informational signals, in laser measuring systems based on a two-mode laser. Specifically, in development of frequency optical standards based on a two-mode He–Ne / CH_4 laser the methods of amplitude and frequency resonances [16] (which involve registration of intensity variation, generating modes and intermode beating frequency in the vicinity of mode tunings to the center of the adsorption line) are usually employed. Accuracy limiting characteristics of such systems are determined by intensity and intermode beating frequency fluctuations [18, 19].

Laser fluctuations are generally divided into technical and natural ones. Technical fluctuations are slower than natural ones, the former being caused by laser parameter instability and therefore can be, in principle, eliminated [20]. But natural fluctuations are associated with quantum properties of radiation and, in contrast to technical fluctuations, cannot be principally eliminated.

Thus the problem of investigation of natural fluctuations in two-mode lasers turns to be fairly important.

The present work contains the study of natural fluctuations of intensity and intermode beating frequency in the two-mode He–Ne and He–Ne / CH_4 lasers with a standing wave generated at the transition $3S_2$–$3P_4$Ne (the wavelength λ is 3.3922 μm).

General regularities describing the behavior of natural fluctuations of laser radiation as opposed to the degree of intermode coupling, spatial mode field separation, mode position in a feedback circuit, intra resonator saturated absorption, etc., are applicable for many types of two-mode gas lasers with inertialess active media.

2. THEORY

Theoretical study of fluctuations of two-mode generation is carried out using an He–Ne laser as an example and adopting the quasi-classical Lamb theory [21] with fluctuation sources which are phenomenologically included.

2.1. Theoretical Calculations of Natural Intensity Fluctuations in a Two-Mode He–Ne Laser

In accordance with the chosen model, the radiation intensity of each mode corresponding to two-mode generation and the presence of an absorbing medium inside the laser resonator can be expressed as

$$\dot{I}_i = 2I_i(\alpha_i^{(+)} + \alpha_i^{(-)} - (\beta_i^{(+)} + \beta_i^{(-)})I_i - (\theta_{ij}^{(+)} + \theta_{ij}^{(-)})I_j) + 2\sqrt{I_i}\omega_0\xi_{ia}(t), \tag{1}$$

where ω_0 is the central frequency of operating transition and $\xi_{ia,\,ir}$, respectively, the sources of intensity and frequency fluctuations [20]. An explicit form of polarizability coefficients for gain ("+") and absorbing ("–") media $\alpha_i^{(\pm)}$, $\beta^{(\pm)}$ and $\theta_{ij}^{(\pm)}$ is presented in [22].

The spectrum width of intensity fluctuation for an He–Ne laser is essentially less than that of fluctuation sources [20, 23]. Thus at different moments of time fluctuation sources in (1) can be assumed not to be correlated, i.e.

$$\langle \xi_{ia}(t)\,\xi_{ja}(t')\rangle = \langle \xi_0^2\rangle k_{ij}\delta(t-t'), \tag{2}$$

where $\langle \xi_0^2\rangle$ is the fluctuation source intensity, k_{ij} is the correlation coefficient for noise sources in ith and jth modes for the same time moment.

In general, fluctuation source intensity depends on operating laser parameters. But in the case of weak fields, $\langle \xi_0^2 \rangle$ can be assumed to be independent on I_i [23] and $\langle \xi_0^2 \rangle$ should be considered a parameter.

Denote average stationary values of mode intensities by $\langle I_i^{(+)} \rangle$ and random components of intensities of the generating modes as $\delta I_i^{(+)}$. If the degree of intensity fluctuations in the laser is much less than the mode intensities

$$\left|\delta I_i^{(+)}(t)\right| \ll \langle I_i^{(+)} \rangle, \qquad (3)$$

then $\delta I_i^{(+)}(t)$ can be taken as small parameters and one can linearize (1) with respect to these parameters. In this case, without accounting for the absorbing medium, one has

$$\begin{aligned} \delta \dot{I}_i^{(+)}(t) &= -2\beta_i^{(+)} \langle I_i^{(+)} \rangle \delta I_i^{(+)}(t) \\ &- 2\theta_{ij}^{(+)} \langle I_i^{(+)} \rangle \delta I_j^{(+)}(t) + 2\sqrt{\langle I_i^{(+)} \rangle}\omega_0 \xi_i(t), \\ &(i, j = 1, 2; i \neq j), \langle I_i^{(+)} \rangle = (\alpha_i^{(+)} \beta_j^{(+)} \\ &- \alpha_j^{(+)} \theta_{ij}^{(+)}) / (\beta_i^{(+)} \beta_j^{(+)} - \theta_{ij}^{(+)} \theta_{ji}^{(+)}). \end{aligned} \qquad (4)$$

Performing the Fourier transformation of (4) and simplifying corresponding expressions the following relationship for spectral density of the intensity of one mode can be obtained [24, 25]:

$$\begin{aligned} \langle \delta I_1^{(+)}(\omega) \rangle^{1/2} &= (J_1/J_2)^{1/2}, \\ J_1 = 4\omega_0 \{ \langle I_1^{(+)} \rangle \omega^2 &+ 4\langle I_1^{(+)} \rangle \langle I_2^{(+)} \rangle^2 \beta_2^{(+)2} \\ + 4\langle I_2^{(+)} \rangle \langle I_1^{(+)} \rangle^2 \theta_{12}^{(+)2} &- 4\langle I_1^{(+)} \rangle^{3/2} \langle I_2^{(+)} \rangle^{3/2} \theta_{12}^{(+)} \omega K_{12}^J \\ -8\langle I_1^{(+)} \rangle^{3/2} \langle I_2^{(+)} \rangle^{3/2} &\beta_2^{(+)} \theta_{12}^{(+)} K_{12}^R \} \langle \xi_{1a}^2 \rangle, \\ J_2 = \{ 4\langle I_1^{(+)} \rangle \langle I_2^{(+)} \rangle &(\beta_1^{(+)} \beta_2^{(+)} - \theta_{12}^{(+)} \theta_{21}^{(+)}) - \omega^2 \} \\ + 4\omega^2 (\beta_1^{(+)} \langle I_1^{(+)} \rangle &+ \beta_2^{(+)} \langle I_2^{(+)} \rangle)^2, \end{aligned} \qquad (5)$$

where $\langle .... \rangle$ denotes statistical averaging

$$K_{12}^J = \frac{\langle \mathrm{Im}(\xi_{1a}(\omega)) \xi_{2a}(\omega) \rangle}{\langle \xi_{1a}^2(\omega) \rangle};$$

$$K_{12}^R = \frac{\langle \mathrm{Re}(\xi_{1a}(\omega)) \xi_{2a}^*(\omega) \rangle}{\langle \xi_{2a}^2(\omega) \rangle}.$$

The relationship for spectral density of intensity fluctuations of the second mode can be deduced from (5) by changing the indices $1 \to 2$ and $2 \to 1$.

An analysis of (5) would argue that the intensity fluctuation spectrum $\langle \delta I_1^{(+)2}(\omega) \rangle^{1/2}$ in a two-mode He–Ne laser is generally non-Lorentz and obeys a more complicated law [25].

2.1.1. Effect of Mode Position on Intensity Fluctuations in a Two-Mode Laser. Let us denote mode detunings from the center of the gain line as ω_{i0} ($\omega_{10} = \omega_{12}/2 + u$; $\omega_{20} = -\omega_{12}/2 + u$, where ω_{12} is the intermode interval, and u the mode detuning from a symmetric position). For a given arrangement of the modes ($\omega_{12} > 0$), let the growth of $u (u > 0)$ moves the first mode away from the center of gain line; in this case denoting the boundaries of the two-mode generation band as u_0 and $(-u_0)$ it is easy to present the spectral density of intensity fluctuations of one of the modes at zero frequency and approaching u_0 as follows:

$$\langle \delta I_1^{(+)2}(\omega = 0) \rangle_{u \to u_0}^{1/2} = \frac{\omega_0 \beta_2^{(+)} \langle \xi_{1a}^2 \rangle^{1/2}}{(\alpha_1^{(+)} \beta_2^{(+)} - \alpha_2^{(+)} \theta_{12}^{(+)})(\beta_1^{(+)} \beta_2^{(+)} - \theta_{12}^{(+)} \theta_{21}^{(+)})}, \qquad (6)$$

and in the case of approaching $(-u_0)$ one finds

$$\langle \delta I_1^{(+)2}(\omega = 0) \rangle_{u \to -u_0}^{1/2} = \frac{\omega_0 \theta_{12}^{(+)} \langle \xi_{1a}^2 \rangle^{1/2}}{(\alpha_2^{(+)} \beta_1^{(+)} - \alpha_1^{(+)} \theta_{21}^{(+)})(\beta_1^{(+)} \beta_2^{(+)} - \theta_{12}^{(+)} \theta_{21}^{(+)})}. \qquad (7)$$

At stable two-mode generation $\beta_2^{(+)} > \theta_{12}^{(+)}$, and

$$(\alpha_1^{(+)} \beta_2^{(+)} - \alpha_2^{(+)} \theta_{12}^{(+)})\big|_{u \to u_0} = (\alpha_2^{(+)} \beta_1^{(+)} - \alpha_1^{(+)} \theta_{21}^{(+)})\big|_{u \to -u_0}.$$

Thus, if the mode approaches (u_0) intensity fluctuations are greater than that for the case of its approaching $(-u_0)$. Such asymmetric behavior of $\langle \delta I_1^{(+)2}(\omega = 0) \rangle^{1/2}$ can be explained by physical distinction of the right (u_0) and the left $(-u_0)$ boundaries of the two-mode generation band. Generation of the first mode diminishes at the right boundary of the two-mode generation band; at the left boundary two-mode generation is terminated due to suppression of the second mode, i.e. in one case the first mode approaches a generation threshold, but in other case it appears to be far away from this threshold; evidentially this ensures distinctions of intensity fluctuation level [24, 25].

An increase of the degree of intermode coupling results in $\beta_i^{(+)}$ and $\theta_{ij}^{(+)}$ becoming closer, i.e. $\theta_{ij}^{(+)} \to \beta_i^{(+)}$ at $S \to 0$. Finally, from the viewpoint of natural intensity fluctuations the right and the left boundaries of the two-mode generation band become equivalent. The dependence of $\langle \delta I_1^{(+)2}(\omega = 0) \rangle^{1/2}$ on u acquires a symmetric form. The maximum of $\langle \delta I_1^{(+)2}(\omega = 0) \rangle^{1/2}$ moves to a symmetric mode position in a gain line circuit. This conclusion is consistent with the results of [26], where it was shown that in the case of tight intermode coupling the dependence of $\langle \delta I_1^{(+)2}(\omega = 0) \rangle^{1/2}$ on u was symmetric with respect to $u = 0$ (the minimum of natural intensity fluctuations coincides with a symmetric mode position in an amplification circuit).

2.1.2. Effect of the Degree of Intermode Coupling on the Level of Intensity Fluctuations at an Arbitrary Mode Position in a Gain Line Circuit. Introduce the parameter S:

$$S = 1/4\,(\beta_1^{(+)}\beta_2^{(+)}/\theta_{12}^{(+)}\theta_{21}^{(+)} - 1), \quad (8)$$

which indicates the degree of intermode coupling [27]. Then the dependence of the level of natural intensity fluctuations on S at zero spectral frequency takes on the form

$$\begin{aligned} &\langle \delta I_1^{(+)2}(\omega = 0)\rangle^{1/2} = A/S, \\ &A = \{\omega_0^2\langle\xi_{1a}^2\rangle[\beta_2^{(+)2}\langle I_2^{(+)}\rangle + \theta_{12}^{(+)2}\langle I_1^{(+)}\rangle \\ &-K_{12}^R\beta_1^{(+)}\theta_{12}^{(+)}\sqrt{\langle I_1^{(+)}\rangle\langle I_2^{(+)}\rangle}\,]/[\langle I_1^{(+)}\rangle\langle I_2^{(+)}\rangle \\ &\times 16\theta_{12}^{(+)2}\theta_{21}^{(+)2}]. \end{aligned} \quad (9)$$

It can be easily seen from (9) that a decrease in S (i.e. enhancing of intermode coupling) leads to an increase in $\langle\delta I_1^{(+)2}(\omega = 0)\rangle^{1/2}$. Fig. 1 displays the calculated dependence of $\langle\delta I_1^{(+)2}(\omega = 0)\rangle^{1/2}$ on S. One can conclude from this figure that for mode detuning from a symmetric position the dependence of $\langle\delta I_1^{(+)2}(\omega = 0)\rangle^{1/2}$ on S becomes more acute. This is connected with the fact that in growth of the degree of intermode coupling for $u \neq 0$ intensity redistribution takes place. Analysis of (9) implies that, in this case, the coefficient A rises, but at $u = 0$ it remains constant. Therefore, at $u \neq 0$ dependence of $\langle\delta I_1^{(+)2}(\omega = 0)\rangle^{1/2}$ on S is steeper than for $u = 0$.

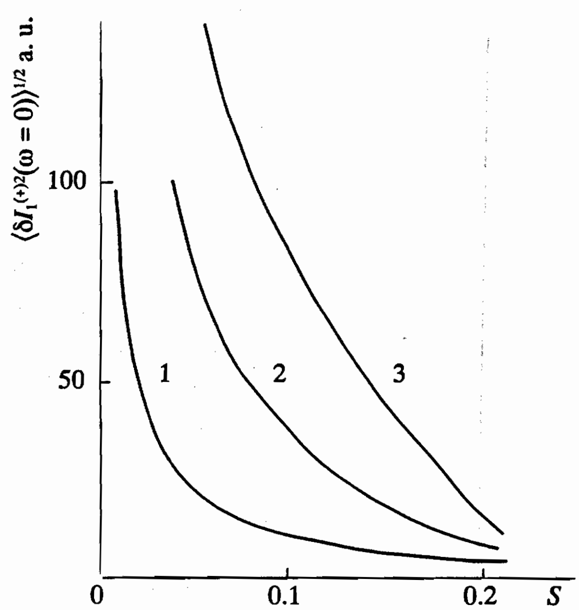

Fig. 1. *Calculated dependence of $\langle\delta I_1^{(+)2}(\omega)\rangle^{1/2}$ on S at ω_{12} = 45 MHz, η = 1.3. Curve 1 corresponds to u = 0 MHz, curve 2 – u = 3 MHz, curve 3 – u = 10 MHz.*

2.2. Natural Fluctuations of Intermode Beating Frequency in a Two-Mode He–Ne Laser

Calculation of intermode beating frequency fluctuations is carried out by solving the following system of equations:

$$\begin{aligned} &\dot{I}_i = 2I_i[(\alpha_i^{(+)} + \alpha_i^{(-)}) - (\beta_i^{(+)} + \beta_i^{(-)})I_i \\ &\times (\theta_{ij}^{(+)} + \theta_{ij}^{(-)})I_j] + 2\sqrt{I_i}\omega_0\xi_{ia}(t), \\ &\omega_i - \Omega_{ip} = (\sigma_i^{(+)} + \sigma_i^{(-)}) + (\rho_i^{(+)} + \rho_i^{(-)})I_i \\ &+ (\tau_{ij}^{(+)} + \tau_{ij}^{(-)})I_j + \omega_0\xi_{ir}(t)/\sqrt{I_i}, \\ &(i,j = 1,2;\ i \neq j), \end{aligned} \quad (10)$$

where ω_i are the frequencies of mode generation, and Ω_{ip} the frequencies of empty resonance modes. An explicit form of the medium polarizability coefficients $\sigma_i^{(\pm)}$, $\rho_i^{(\pm)}$ and $\tau_{ij}^{(\pm)}$ is contained in [22, 27].

Represent the solution of (10) for frequency as

$$\omega_i^{(+)}(t) = \langle\omega_i^{(+)}\rangle + \delta\omega_i^{(+)}(t),$$

where $\langle\omega_i^{(+)}\rangle$ is the average stationary value, and $\delta\omega_i^{(+)}(t)$ the stochastic frequency fluctuations. Denote fluctuations of intermode beating frequency as $\delta\psi^{(+)} = \delta\omega_1^{(+)}(t) - \delta\omega_2^{(+)}(t)$. In practice, as a rule, the condition $|\delta\omega_i^{(+)}(t)| \ll \langle\omega_i^{(+)}\rangle$ is satisfied for gas lasers. Performing linearization of (10) with respect to small parameters $\delta I_i^{(+)}(t)$ and $\delta\omega_i^{(+)}(t)$, one finds

$$\begin{aligned} &\delta\dot{I}_i^{(+)} = -2\beta_i^{(+)}\langle I_i^{(+)}\rangle\delta I_i^{(+)} \\ &-2\theta_{ij}^{(+)}\langle I_j^{(+)}\rangle\delta I_j^{(+)} + 2\omega_0\sqrt{\langle I_i^{(+)}\rangle}\xi_{ia}(t), \\ &\delta\omega_i^{(+)}(t) = \rho_i^{(+)}\delta I_i^{(+)} + \tau_{ij}^{(+)}\delta I_j^{(+)} \\ &+ \omega_0\xi_{ir}(t)/\sqrt{\langle I_i^{(+)}\rangle}. \end{aligned} \quad (11)$$

2.2.1. Case of Symmetric Mode Position in a Gain Line Circuit. Consider for the sake of simplicity the case of a symmetric mode position in a gain line circuit.

Performing the Fourier transformation of (11) and corresponding calculations, one can obtain an expression for the square of the spectral density of intermode beating frequency fluctuations and the square of frequency fluctuations of a single mode [28, 29]

$$\langle\delta\dot{\psi}^2\rangle = (\rho^{(+)} + \tau^{(+)})^2 \langle \delta I^{(+)2}(\omega)\rangle + \frac{2\omega_0}{\langle I^{(+)}\rangle}\langle\xi_{1r}^2\rangle(1-K_r) + \frac{(\rho^{(+)}+\tau^{(+)})\,\omega_0}{\langle I^{(+)}\rangle^{1/2}} \times (\delta I^{(+)}(\omega)\,(\xi_{1r}^* - \xi_{2r}^*) + \langle \delta I^{(+)*}(\omega)\,(\xi_{1r} - \xi_{2r})\rangle, \quad (12)$$

$$\langle\dot{\varphi}^2(\omega)\rangle = \rho^{(+)2}\langle\delta I_1^{2(+)}(\omega)\rangle + \tau^{(+)2}\langle\delta I_2^{(+)2}(\omega)\rangle + \rho^{(+)}\tau^{(+)}2\mathrm{Re}\,\langle\delta I_1^{(+)}(\omega)\rangle + \frac{\omega_0^2}{\langle I^{(+)}\rangle}\langle\xi_{1r}^2\rangle + \frac{\rho^{(+)}\omega_0}{\langle I^{(+)}\rangle^{1/2}}2\mathrm{Re}\,\langle\delta I_1^{(+)}(\omega)\,\xi_1^*(\omega)\rangle + \frac{\tau^{(+)}\omega_0}{\langle I^{(+)}\rangle^{1/2}}2\mathrm{Re}\,\langle\delta I_2^{(+)}(\omega)\,\xi_1^*(\omega)\rangle, \quad (13)$$

where $\delta I^{(+)} = \delta I_1^{(+)} + \delta I_2^{(+)}$ is the fluctuation of total radiation intensity.

$$\delta I^{(+)}(\omega) = \frac{2\langle I^{(+)}\rangle^{1/2}\omega_0(\xi_{1a}+\xi_{2a})}{(i\omega + 2\alpha^{(+)})}.$$

One can see from (12) that $\langle\delta\dot{\psi}^2\rangle$ consists of three terms. The first term is connected with the influence of total intensity fluctuations of two modes on $\langle\delta\dot{\psi}^2\rangle$. It takes the form of a Lorentz line with width $2\alpha^{(+)}$.

The second term is associated with frequency fluctuation sources. Two factors determine the spectrum of frequency fluctuation sources: (i) relaxation time of polarization fluctuations of the active medium (specific times are of the order of $\sim 1/\gamma_a$, $1/\gamma_b$, $1/\gamma$) and; (ii) relaxation of thermal fluctuations of an empty resonator (relaxation times of thermal fluctuations are about $\sim 1/\Delta\omega_p$). For a typical two-mode He–Ne laser (λ = 3.39 μm) the width of the resonance line is of the order of 10 - 100 MHz, but the widths of the levels and lines exceed 10 MHz at the pressure of the active medium ≅ 2 torr. Consequently, in the interval of 0 - 1 MHz, the second term can be adequately approximated by a constant. The third term attributes interactions of purely frequency fluctuation sources with intensity fluctuation sources. If one assumes that intensity and frequency fluctuation sources in a two-mode laser are not correlated (i.e. $(\langle\xi_{ia}(t)\xi_{jr}(t)\rangle = 0)$ as already shown for a one-mode laser [20], then the third term can be ignored.

The spectrum of frequency fluctuations of a single mode $\langle\delta\dot{\varphi}_1^2(\omega)\rangle$ reveals a somewhat different form. The first two terms represent the influence of intensity fluctuations of each mode on frequency fluctuations. They adopt a Lorentz form with width $2\alpha^{(+)}(\beta^{(+)} - \theta^{(+)}) / (\beta^{(+)} + \theta^{(+)})$. The third term appears due to interactions of intensity fluctuations of single modes. The fourth term attributes purely frequency noises, and the fifth represents the interaction of frequency and intensity fluctuations.

As one can see from (12, 13), for two-mode generation for tight intermode coupling $((\beta^{(+)} - \theta^{(+)}) / (\beta^{(+)} + \theta^{(+)}) \ll 1)$ the influence of intensity fluctuations on $\langle\delta\dot{\varphi}_1^2(\omega=0)\rangle$ is much greater than on $\langle\delta\dot{\psi}^2(\omega=0)\rangle$. At the same time the contribution of frequency fluctuations to $\langle\delta\dot{\psi}^2(\omega=0)\rangle$ for $K = -1$ is four times greater than that in $\langle\delta\dot{\varphi}_1^2(\omega=0)\rangle$; but at $K = 1/2$ or 1 this contribution is the same or essentially less.

In order to compare the contribution from intensity and frequency fluctuation sources to frequency fluctuations of intermode beating and single mode we suggest that $\langle\xi_{1a}^2\rangle = \langle\xi_{1r}^2\rangle$. For the case of weak fields this assumption is quite natural [20]. Then using (12 one is able to show that the ratio of the contributions from purely frequency fluctuation sources to intensity fluctuation sources to $\langle\delta\dot{\psi}^2(\omega)\rangle$ is given by the expression

$$\frac{\text{freq.}}{\text{int.}} \sim \frac{\gamma}{\omega_{12}}(1-K_r). \quad (14)$$

An analogous ratio for $\langle\delta\dot{\varphi}_1^2(\omega=0)\rangle$ takes on the form

$$\frac{\text{freq.}}{\text{int.}} \sim \frac{\gamma}{\omega_{12}}\frac{(\beta^{(+)} - \theta^{(+)})}{(\beta^{(+)} + \theta^{(+)})}. \quad (15)$$

Equation (14) shows that if $\omega_{12} \ll \gamma$ and $K_r = 0$ or -1, then frequency fluctuations of intermode beating are primarily determined by purely frequency noises. At $\omega_{12} \gtrsim \gamma$ or $K_r = 1$ the contribution from intensity fluctuations to frequency fluctuations of intermode beating can be commensurate with purely frequency noises. But one can see from (15) that if the condition $\gamma < \omega_{12}(\beta^{(+)} + \theta^{(+)}) / (\beta^{(+)} - \theta^{(+)})$ holds in a two-mode laser, then the spectrum of frequency fluctuations of a single mode is specified by the influence of intensity fluctuations on frequency. At $\gamma > \omega_{12}(\beta^{(+)} + \theta^{(+)}) / (\beta^{(+)} - \theta^{(+)})$ the spectrum of frequency fluctuations of a single mode will be determined by purely frequency noises.

Comparing (14) and (15) one finds that in the case of strong intermode coupling the level of frequency fluctuations of intermode beating is much less than frequency fluctuations of a single mode at $K_r < 1$.

It is possible to obtain the contribution of amplitude and frequency noises to frequency fluctuations of intermode beating by studying the dependence of $\langle\delta\dot{\psi}^2(\omega)\rangle$ on operating laser parameters.

As clearly seen from (12) the first and second terms depend in a different way on intermode interval and spatial shift between mode fields. A growth in ω_{12}, $(\rho^{(+)} + \tau^{(+)})^2$ results in increase of ω_{12}^2. The intensity $\langle I^{(+)}\rangle$ weakly depends on ω_{12}. Therefore an increase of ω_{12} is accompanied by a growth of the first term in (12); at the same time the second term remains practically

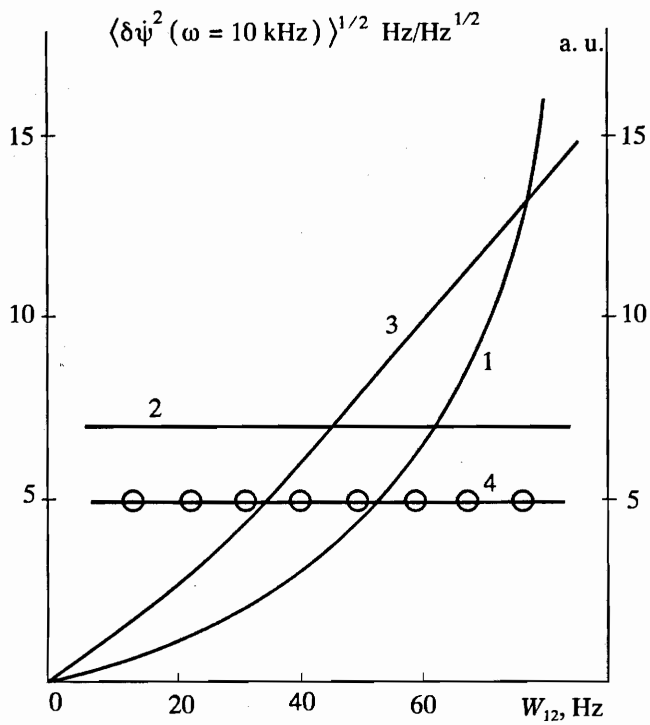

Fig. 2. *Calculated and experimental dependence of $\langle \delta\dot\psi^2 (\omega = 10\ kHz) \rangle^{1/2}$ on ω_{12}: first term (1), second term (2), third term (3) at $\delta = 60°$.*

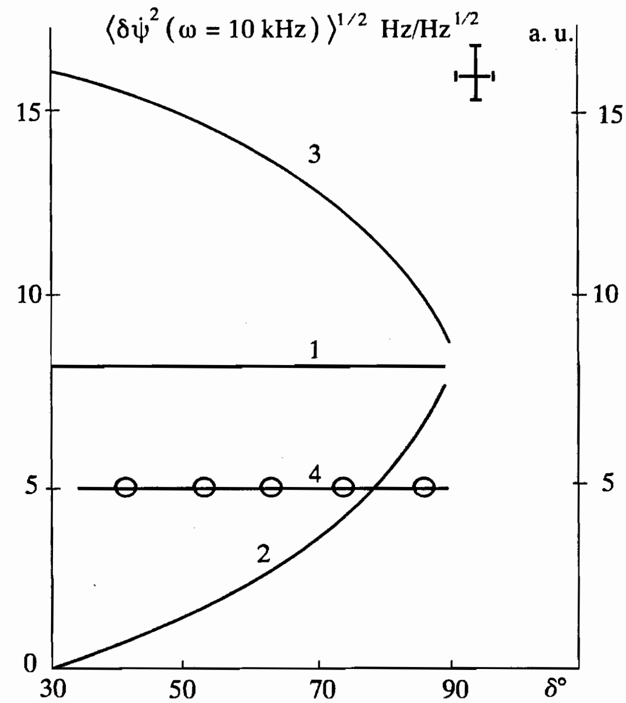

Fig. 3. *Qualitative theoretical (1, 2, 3) and experimental (4) dependences of $\langle \delta\dot\psi^2 (\omega = 10\ kHz) \rangle^{1/2}$ on δ: (1) – $K_r = 0$, (2) – $K_r = 1$, (3) – $K_r = -1$ at $\omega_{12} = 40$ MHz.*

constant. The third term grows as ω_{12} (Fig. 2, curves 1, 2, 3).

It is shown in [30] that, in the case of a symmetric mode position in a gain line circuit, the spatial shift δ weakly affects $\langle I^{(+)} \rangle$, $\tau^{(+)}$ and $\rho^{(+)}$. Therefore, the first term in (12) weakly depends on δ.

The second term may depend on δ more heavily if ξ_{1r} correlates with ξ_{2r} (i.e. $K_r = 1$) or anticorrelates with ξ_{2r} ($K_r = -1$), because at spatial separation of mode fields, noise source correlation must approach zero (obviously, if this correlation does occur). At $K_r = 1$ the dependence of $\langle \delta\dot\psi^2 (\omega = 0) \rangle$ on δ is enhanced (Fig. 3, curve 2), because frequency noise sources at spatial shift must become statistically independent. At $K_r = 1$ the dependence of $\langle \delta\dot\psi^2 (\omega = 0) \rangle$ on δ declines (see Fig. 3, curve 3), and at $K_r = 0$ it remains constant (see Fig. 3, curve 1). It should be noted that the dependences of $\langle \delta\dot\psi^2 (\omega = 0) \rangle$ on δ (curves 1, 2, 3 in Fig. 3) are shown qualitatively because the dependences of $\langle \xi_{1r}^2 \rangle$ and Re $\langle \xi_{1r} \xi_{2r}^* \rangle$ on δ are unknown.

With a growth in the excess η the first and second terms in (12) decrease in a similar manner. Accordingly, in any case with a growth in η $\langle \delta\dot\psi^2 (\omega = 0) \rangle$ is to be decreased.

Thus, the theoretical treatment of $\langle \delta\dot\psi^2 (\omega) \rangle$ presented here argues that three sources of intermode beating frequency fluctuations for the case of a symmetric mode position in a gain line circuit are possible: (i) fluctuation of total intensity of two modes; (ii) purely frequency noise sources and; (iii) interaction of fluctuation sources of total intensity with purely frequency noise sources. One can find the particular contribution of each source of intermode beating frequency fluctuations by studying the dependence of $\langle \delta\dot\psi^2 (\omega) \rangle$ on operating laser parameters, i.e. intermode interval and spatial shift [28, 29].

2.2.2. Arbitrary Mode Position in a Gain Line Circuit. For the case of an arbitrary mode position in a gain line circuit one has $\rho_1^{(+)} \neq -\rho_2^{(+)}$; $\tau_{12}^{(+)} \neq -\tau_{21}^{(+)}$. Performing the Fourier transformation of (11) and corresponding calculations, one can derive the relationship for the spectral density of intermode beating frequency fluctuations

$$\begin{aligned}\langle \delta\dot\psi^2 (\omega) \rangle = {} & (\rho_1^{(+)} - \tau_{12}^{(+)})^2 \langle \delta I_1^{(+)2} (\omega) \rangle \\ & + (\tau_{12}^{(+)} - \rho_2^{(+)})^2 \langle \delta I_2^{(+)2} (\omega) \rangle \\ & + (\rho_1^{(+)} - \tau_{21}^{(+)}) (\tau_{12}^{(+)} - \rho_2^{(+)}) \\ & \times 2\mathrm{Re} \langle \delta I_2^{(+)} (\omega) \, \delta I_1^{(+)*} (\omega) \rangle + \omega_0^2 \langle \xi_F^2 \rangle \\ & + \omega_0 (\rho_1^{(+)} - \tau_{21}^{(+)}) \, 2\mathrm{Re} \langle \xi_F \delta I_1^{(+)*} (\omega) \rangle \\ & - \omega_0 (\rho_2^{(+)} - \tau_{12}^{(+)}) \, 2\mathrm{Re} \langle \xi_F \delta I_2^{(+)*} (\omega) \rangle,\end{aligned} \tag{16}$$

where

$$\xi_F = \xi_{1r}(\omega)\langle I_2^{(+)}\rangle^{1/2} + \xi_{2r}(\omega)\langle I_1^{(+)}\rangle^{1/2};$$

$$\langle \delta I_{1,2}^{(+)2}(\omega)\rangle$$

are defined by the expression (9) and $\mathrm{Re}\langle\delta I_2(\omega)\delta I_1^*(\omega)\rangle$ is of the form

$$\begin{aligned}\mathrm{Re}\langle\delta I_2^{(+)}(\omega)\,\delta I_1^{(+)*}(\omega)\rangle = (\{4\langle I_1^{(+)}\rangle^{1/2}\\ \times\langle I_2^{(+)}\rangle^{1/2}\omega_0^2\omega^2 + 16\langle I_2^{(+)}\rangle^{1/2}\omega_0^2(\beta_1^{(+)}\beta_2^{(+)}\\ +\theta_{12}^{(+)}\theta_{21}^{(+)})\}\,\mathrm{Re}\langle\xi_{2a}\xi_{1a}\rangle - 16\langle I_1^{(+)}\rangle\langle I_2^{(+)}\rangle\\ \times\omega_0^2\langle|\xi_1^2|\rangle(\theta_{21}^{(+)}\beta_2^{(+)}\langle I_2^{(+)}\rangle + \theta_{12}^{(+)}\beta_1^{(+)}\langle I_1^{(+)}\rangle)\\ -8\langle I_1^{(+)}\rangle^{1/2}\langle I_2^{(+)}\rangle^{1/2}\omega_0^2(\langle I_2^{(+)}\rangle\beta_2^{(+)} - \langle I_1^{(+)}\rangle\beta_1^{(+)})\\ \times\omega\,\mathrm{Im}\langle\xi_{2a}\xi_{1a}^*\rangle)/(\{4\langle I_1^{(+)}\rangle\langle I_2^{(+)}\rangle(\beta_1^{(+)}\beta_2^{(+)}\\ -\theta_{12}^{(+)}\theta_{21}^{(+)}) - \omega^2\}^2 + 4\omega^2(\beta_1^{(+)}\langle I_1^{(+)}\rangle\\ +\beta_2^{(+)}\langle I_2^{(+)}\rangle)^2).\end{aligned} \quad (17)$$

In the case of a symmetric mode position instead of (16) one has the corresponding (12) obtained in the previous section.

The first three terms in (16) attribute the effect of intensity fluctuations on natural fluctuations of intermode beating. In the case of a symmetric mode position, intensity fluctuations of a single mode are mutually compensated. Therefore intermode beating frequency fluctuations depend only on natural fluctuations of total intensity. Natural intensity fluctuations of a single mode produce no influence on $\langle\delta\psi^2(\omega)\rangle$. For detuning of the modes from a symmetric position, natural intensity fluctuations of the first and second modes behave differently. As a result, the effect of natural intensity fluctuations of the modes on natural fluctuations of intermode beating frequency turns out to be unbalanced. Natural fluctuations of intermode beating frequency begin to depend on natural intensity fluctuations of a single mode. In the case of strong intermode coupling, the level of spectral density of natural intensity fluctuations for a single mode is much higher than the total intensity fluctuations. For this reason, detuning of modes from a symmetric position makes the influence of natural intensity fluctuations on natural fluctuations of intermode beating frequency stronger. The term $\delta\psi$, which expresses the influence of intensity fluctuations on natural fluctuations of intermode beating frequency, can be expressed as the sum of three spectra (for fluctuations of intensity of a single mode and total intensity).

The fourth term in (16) is purely caused by frequency noises. Similar for the case of a symmetric mode position in a gain line circuit its spectrum profile can be adequately approximated by a constant at $\omega < 200$ kHz.

If detuning of modes from a symmetric position with respect to the center of a gain line circuit takes place, its contribution to (16) also grows. This happens due to intensity redistribution between the modes. For the case of tight intermode coupling the intensities of the modes linearly depend on u:

$$\langle I_{1,2}^{(+)}\rangle = I_{\mathrm{sum}}/2 \mp uB. \quad (18)$$

Transforming the fourth term with (18) in mind, one obtains the fourth term:

$$\backsim \mathrm{const}\,(1 + 4B^2u^2/I_{\mathrm{sum}}^2) \quad (19)$$

near detuning of the modes from a symmetric position in a gain line circuit, i.e. with detuning of modes from a symmetric position an increase in the contribution of purely frequency noises to natural fluctuations of intermode beating frequency obeys a square law. The fifth and sixth terms in (16) are associated with a correlation between sources of intensity fluctuations and frequency fluctuations of a single mode.

Analysis conducted here demonstrates that the total level of natural fluctuations of intermode beating frequency at detuning of the modes from a symmetric position should grow because of an increase in the absolute contribution of intensity fluctuations and purely frequency fluctuations to natural fluctuations of intermode beating frequency. A possible way to evaluate the relative contribution of natural intensity fluctuations and purely frequency fluctuations to natural fluctuations of intermode beating frequency at various u is to compare theoretical dependences of $\langle\delta\psi(\omega = \omega')\rangle^{1/2}$ on u with experimental ones.

In addition, one can distinguish contributions of natural intensity fluctuations and purely frequency noises to natural fluctuations of intermode beating frequency at detuning of the modes from a symmetric position by analyzing the spectrum profile of $\langle\delta\psi^2(\omega)\rangle$ (because the level of spectral density of natural intensity fluctuations constantly decreases and the contribution of purely frequency terms to (16) does not depend on ω). Moreover, by enhancing the degree of intermode coupling one can reduce the width of the natural intensity fluctuations spectrum up to dozens of kHz, while the spectrum profile of the purely frequency noise source remains constant [29].

3. EFFECT OF NON-LINEAR INTRARESONATOR SATURATED ABSORPTION ON NATURAL INTENSITY FLUCTUATIONS AND NATURAL FLUCTUATIONS OF INTERMODE BEATING FREQUENCY IN TWO-MODE LASERS

3.1. Non-Linear Resonances of Natural Intensity Fluctuations in Two-Mode He–Ne / CH_4 Lasers

A theoretical investigation of intensity fluctuations in two-mode lasers with inner absorbing cell is carried out using (1). We assume that this cell introduces no additional contribution to the sources of natural intensity fluctuations.

Linearizing (1) with respect to the small parameters δI_i and performing the Fourier transformation of the linearized system it is straightforward to obtain the expression for the spectral density of intensity fluctuations of the first mode. It coincides with (9), if the following procedure

$$\alpha_i^{(+)} \to \alpha_i^{(+)} + \alpha_i^{(-)}; \beta_i^{(+)} \to \beta_i^{(+)} + \beta_i^{(-)};$$
$$\theta_{ij}^{(+)} \to \theta_{ij}^{(+)} + \theta_{ij}^{(-)}$$

is done. In this case, the level of the spectral density of natural intensity fluctuations of the first mode at zero spectral frequency is

$$\langle \delta I_1^2(\omega = 0)\rangle^{1/2} = \{(\omega_0\langle\xi_{1a}^2\rangle[(\beta_2^{(+)} + \beta_2^{(-)})^2\langle I_2\rangle + (\theta_{12}^{(+)} + \theta_{12}^{(-)})\langle I_1\rangle - 2K_{12}^R(\beta_2^{(+)} + \beta_2^{(-)})(\theta_{12}^{(+)} + \theta_{12}^{(-)})\sqrt{\langle I_1\rangle\langle I_2\rangle}])/(\langle I_1\rangle\langle I_2\rangle \times S^2 16(\theta_{12}^{(+)} + \theta_{12}^{(-)})^2(\theta_{21}^{(+)} + \theta_{21}^{(-)})^2)\}^{1/2} \quad (20)$$

$$\langle I_i\rangle = [(\alpha_i^{(+)} + \alpha_i^{(-)})(\beta_j^{(+)} + \beta_j^{(-)}) - (\alpha_j^{(+)} + \alpha_j^{(-)})(\theta_{ij}^{(+)} + \theta_{ij}^{(-)})]/[(\beta_i^{(+)} + \beta_i^{(-)})(\beta_j^{(+)} + \beta_j^{(-)}) - (\theta_{ij}^{(+)} + \theta_{ij}^{(-)})(\theta_{ji}^{(+)} + \theta_{ji}^{(-)})]$$

(see [31, 32]). In the framework of our model it is impossible to calculate K_{12}^R; for this reason, we suggest that its parameters vary from -1 to 1, i.e. $K_{12}^R \in [-1, 1]$. In the presence of a non-linear absorber inside the resonator the parameter S which specifies the degree of intermode coupling in the case of an arbitrary mode position in a gain line takes the form

$$S = 1/4 \frac{(\beta_1^{(+)} + \beta_1^{(-)})(\beta_2^{(+)} + \beta_2^{(-)})}{(\theta_{12}^{(+)} + \theta_{12}^{(-)})(\theta_{21}^{(+)} + \theta_{21}^{(-)})}^{-1}. \quad (21)$$

In order to analyze $\langle\delta I_1^2(\omega = 0)\rangle^{1/2}$ near tuning of the modes to the center of the absorption line let us present an explicit form of the coefficients $\beta_1^{(-)}$ and $\theta_{12}^{(-)}$, neglecting non-resonance terms in the case of coincidence of the centers of gain and absorption lines

$$\beta_1^{(-)} = \frac{\alpha_1^{(-)}d^{(-)2}}{2\hbar^2\gamma^{(-)2}}\frac{\gamma^{(-)2}}{\gamma^{(-)2} + (\omega_{12/2} + u)^2};$$

$$\theta_{12}^{(-)} = \frac{\alpha_1^{(-)}d^{(-)2}}{2\hbar^2\gamma^{(-)2}}\frac{\gamma^{(-)2}}{\gamma^{(-)2} + u^2},$$

where $d^{(-)}$ is the dipole transition momentum, $\gamma^{(-)}$ is the width of the methane absorption line, and $\hbar$ is the Planck's constant.

It is seen from (20) that intensity fluctuations in a two-mode laser depend on mode intensities and intermode coupling S. It is known that the presence of an intraresonator non-linear absorbing cell gives rise to intensity resonances at mode tuning to the center of the absorption line and a symmetric mode tuning with respect to the center of the absorption line. In this case a resonant growth of intensity of one mode is accompanied by a simultaneous resonant decrease of intensity of the second mode. As one can obtain from (20), $\langle I_1\rangle$ and $\langle I_2\rangle$ are contained in $\langle\delta I_1^2(\omega = 0)\rangle^{1/2}$ as a combination $\langle I_1\rangle\langle I_2\rangle$ and $\beta_2^{(+)2}\langle I_2\rangle + \theta_{12}^{(+)2}\langle I_1\rangle$. At small amplitudes of intensity resonances these terms weakly depend on detuning near the center of the absorption line.

Consequently, a change in mode intensities in the interval of intensity resonances will exert weak natural intensity fluctuations.

Fig. 4 (curve 4) displays calculated dependence of S on u. The following values of laser parameters to be studied are assigned for our computations: the length of

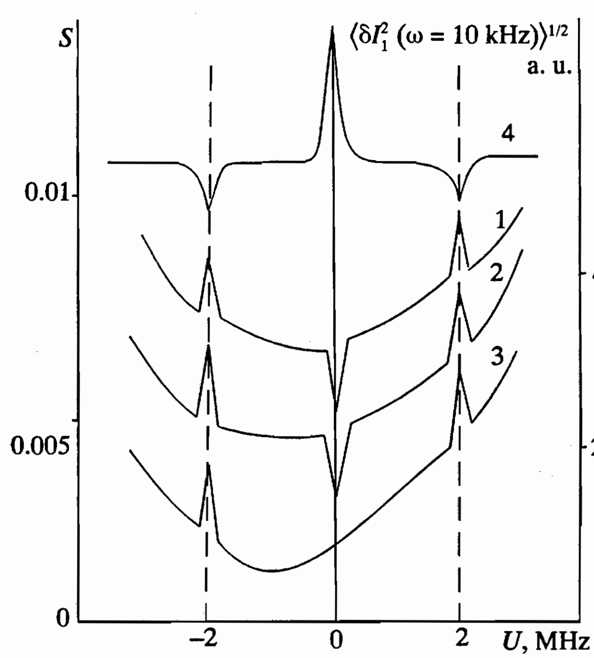

Fig. 4. *Calculated dependence of S on u (curve 4) and calculated dependence of* $\langle\delta\psi^2(\omega = 10\,kHz)\rangle^{1/2}$ *on u (2, 3, 4) for various* K_{12}^R *: (1)* $-K_{12}^R = -1$, *(2)* $-K_{12}^R = 0$, *(3)* $-K_{12}^R = 0.7$ *at* $\delta = 85°$.

the amplifying tube is 0.35 m with capillary diameter 3 mm; the gas mixture pressure is equal to 2 torr, the ratio of partial pressures of He : Ne is 20 : 1; the absorbing cell of length 0.3 m is filled with CH_4 under the pressure of 0.5 mtorr; and the laser resonator of length 1 m is formed by the plane mirror and the mirror with radius of curvature 2 m, if power transmission factors are 2 and 18%, respectively.

We choose the following values of active medium parameters: A = 30 MHz/torr, $A_b^{(2)}$ = 5 MHz/torr, $A_a^{(2)}$ = (20 - 30) MHz/torr, $ku \cong 170$ MHz, $\gamma_b^{(0)}$ = 10 MHz, $\gamma_a^{(2)}$ = 10 MHz, ω_{12} = 10 MHz, η = 1.3; $p^{(+)}$ = 2 torr, $\omega_0 / 2Q$ = 10 MHz.

Fig. 4 clearly shows that, if the mode coincides with the center of the absorption line the parameter S decreases, i.e. the degree of intermode coupling increases. In the case of a symmetric mode position relative to the center of the absorption line S grows, i.e. the degree of intermode coupling declines. In turn, as one can conclude from (20) this leads to the formation of three resonant structures in the dependence of natural intensity fluctuations of a single mode on detuning (these structures correspond to tuning of each mode to the center of the absorption line ($u = \pm\,\omega_{12} / 2$) and symmetric mode position with respect to the center of the absorption line $u = 0$).

The calculated dependence of $\langle\delta I_1^2(\omega = 0)\rangle^{1/2}$ on u at various values of K_{12}^R is plotted in Fig. 4. Quality factors of the modes are assumed to be equal. For calculations we choose the same values for the parameters of active and absorbing media as for calculation of S. It is apparent that at any K_{12}^R there are two narrow resonances in the dependence of $\langle\delta I_1^2(\omega = 0)\rangle^{1/2}$ on u which are directed up and correspond to resonances of the functions $\beta_1^{(-)}$ and $\beta_2^{(-)}$. If mode noise sources are anticorrelated ($K_{12}^R = -1$) (curve 2) or weakly correlated ($0 \lesssim K_{12}^R \ll 1$) (curve 3), then for a symmetric mode position with respect to the absorption line the third resonance directed down (due to a decrease in interaction between the modes) arises. For strong correlation between mode noise sources ($K_{12}^R \cong [0.8 - 0.9]$) the third resonance at a given mode tuning does not appear in the dependence of $\langle\delta I_1^2(\omega = 0)\rangle^{1/2}$ on u (Fig. 4, curve 3). In the experiments the third resonance directed down does not occur. According to analysis of (20) this bears witness to a strong correlation of noise sources in the modes. Let us confine ourselves to the treatment of behavior of $\langle\delta I_1^2(\omega = 0)\rangle^{1/2}$ in the vicinity of the point $u = -\omega_{12} / 2$. In this case $\theta_{12}^{(-)}$ and $\beta_2^{(-)} \ll \beta_1^{(-)}$ With this in mind one yields from (20) [31, 32]

$$\langle\delta I_1^2(\omega = 0)\rangle^{1/2} = \langle\delta I_1^{(+)2}(\omega = 0)\rangle^{1/2} - \frac{\beta_2^{(+)}\beta_1^{(-)}}{\beta_2^{(+)}\beta_1^{(+)} - \theta_{12}^{(+)}\theta_{21}^{(+)}}\langle\delta I_1^{(+)2}(\omega = 0)\rangle^{1/2}, \qquad (22)$$

where

$$\langle\delta I_1^{(+)2}(\omega = 0)\rangle^{1/2} = \omega_0\left(\frac{\beta_2^{(+)2}}{\langle I_1^{(+)}\rangle} + \frac{\theta_{12}^{(+)2}}{\langle I_2^{(+)}\rangle} - \frac{2K_{12}^R\beta_2^{(+)}\theta_{12}^{(+)}}{\sqrt{\langle I_1^{(+)}\rangle\langle I_2^{(+)}\rangle}}\right)^{1/2}\frac{\langle\xi_{1a}^2\rangle^{1/2}}{\beta_1^{(+)}\beta_2^{(+)} - \theta_{12}^{(+)}\theta_{21}^{(+)}}.$$

Equation (22) is valid for a small contrast range of intensity resonances. It implies that the presence of the absorbing medium imparts an increase in natural intensity fluctuations of the first mode in the interval of its tuning to the center of the absorption line in comparison with the value $\langle\delta I_1^{(+)2}(\omega = 0)\rangle^{1/2}$. This is associated with the fact that at a given mode tuning the contribution of the absorbing medium to mode interaction ensures a growth in intermode coupling. An analogous effect can be observed at tuning of the second mode to the center of the absorption line.

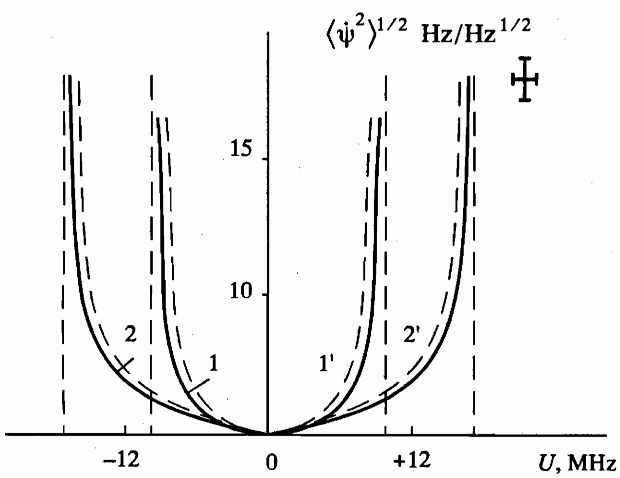

Fig. 5. *Experimental dependence of $\langle\delta\dot{\psi}^2(\omega =$ 10 kHz$)\rangle^{1/2}$ on u at δ = 60° for various ω_{12}: (1) – ω_{12} = 10 MHz, (2) – ω_{12} = 45 MHz.*

3.2. Effect of Methane on Natural Fluctuations of Intermode Beating Frequency

In general, it is difficult to theoretically account for the effect of intraresonator saturated absorption on natural fluctuations of intermode beating frequency. We confine ourselves to the case the first mode occurs near the center of the absorption line and the second one is detuned in such a manner that the influence of methane on this mode can be ignored. In the case of weak intermode coupling ($\tau_{12,\,21}^{(\pm)} \cong 0$) the equations describing generation frequency fluctuations of the first ω_1 and the second ω_2 modes can be written as follows [32, 33]

$$\omega_1 - \Omega_{1p} = \sigma_1^{(+)} + \rho_1^{(+)}E_1^2 + \sigma_1^{(-)} + \rho_1^{(-)}E_1^2 + G(t) + \omega_0\xi_{1r}(t)/E_1; \qquad (23)$$

$$\omega_2 - \Omega_{2p} = \sigma_2^{(+)} + \rho_2^{(+)}E_2^2 + \omega_0\xi_{2r}(t)/E_2,$$

where

$$G(t) = \frac{A_{(-)}}{E_1}\int_0^t d\tau' l^{-\gamma_0^{(-)}\tau}\int_0^{(t-\tau)} d\tau' l^{-2\gamma^{(-)}\tau'}\sin(2\omega_{10}\tau')$$
$$\times E_1(t-\tau')E_1(t-2\tau'-\tau)E(t-2\tau') - \rho_1^{(-)}E_1^2$$

is the function which accounts for the lag properties of methane at generation frequency fluctuations (because the homogeneous widths of absorption lines and operating levels of methane are essentially less than the width of the resonator band); $\gamma_0^{(-)}$ is the width of the methane operating level (for simplicity, in deriving the expression for $G(t)$ the widths of upper and lower levels for methane are taken to be equal [34]; $A_{(-)}$ is the parameter proportional to methane pressure which depends on its spectroscopic characteristics. To be more specific, mean square values of frequency noise sources for the first and second modes are assumed equal

$$\langle \xi_{1r}^2(t=0)\rangle = \langle \xi_{2r}^2(t=0)\rangle = \langle \xi_r^2(t=0)\rangle.$$

Noise sources in (23) are supposed to be weakly correlated at very short times, i.e.

$$\langle \xi_{ir}(t)\xi_{jr}(0)\rangle = \delta_{ij}\langle \xi_r^2(0)\rangle l^{-Dt},$$

where D is the diffusion coefficient specifying the spectral band of noise sources [23] (the value of this band is much greater than that of the resonator band).

Ignoring the influence of intensity fluctuations on frequency fluctuations one obtains from (23)

$$\delta\psi(t) = \frac{\omega_0\xi_{1r}(t)}{E_1} - \frac{\omega_0\xi_{2r}(t)}{E_2} + \frac{\omega_0\xi_{1r}(t)}{E_1}\frac{\partial}{\partial\omega_{10}}G(t), \quad (24)$$

where $\delta\psi(t) = \delta\omega_1(t) - \delta\omega_2(t)$.

Performing the Fourier transformation of (24) and corresponding averaging procedures one can show that the spectral density $\delta\psi(t)$ at $\omega = 0$ near the center of the absorption line takes on the form [32, 33, 35]

$$\langle\delta\psi^2(\omega=0)\rangle^{1/2} = \Big\{\frac{\omega_0^2\langle\xi_r^2(0)\rangle}{D}\frac{E_1^2+E_2^2}{E_1^2E_2^2} + \frac{\omega_0^2\langle\xi_r^2(0)\rangle A_{(-)}(\gamma^{(-)}-\gamma_0^{(-)})}{D^3} \times \frac{\left[(\gamma^{(-)}-\gamma_0^{(-)})^2-\omega_{10}^2\right]}{\left[(\gamma^{(-)}-\gamma_0^{(-)})^2+\omega_{10}^2\right]^2}\Big\}^{1/2}. \quad (25)$$

The first term in (25) is determined by mode intensities. If mode intensities coincide with methane they follow the law [16]

$$E_{1,2}^2 = E_0^2\{1/2 + (K\left[\tilde\beta_1^{(-)}(u) - \tilde\beta_2^{(-)}(u)\right] - u\tau)\},$$

where $E_0^2 = E_1^2 + E_2^2 = \text{const}$ is the total mode intensity,

$$\tau = 1/2\frac{\beta+\theta}{\beta-\theta}\left(\frac{\alpha^{(1)}}{\alpha_0} + \frac{\beta^{(1)}+\theta^{(1)}}{\beta-\theta}\right)$$

the coefficients $\alpha^{(1)}$, $\beta^{(1)}$, $\theta^{(1)}$ at expansion terms of the magnitudes α_k, β_k, θ_{kl} linear with respect to u [22], and the contrast range of intensity resonances (being equal to the ratio of resonance source value to total mode intensity). In this case the first term in (25) depends on mode detuning from the center of the absorption line (the center of the absorption line coincides with the center of a gain line u) as

$$\mathcal{L} = \frac{E_1^2+E_2^2}{E_1^2E_2^2}\frac{\omega_0^2\langle\xi_r^2(0)\rangle}{D} \cong \{1/4 - 2K(\tilde\beta_1^{(-)}(u) - \tilde\beta_2^{(-)}(u))u + u^2\tau\}\,\omega_0^2\langle\xi_r^2(0)\rangle/D. \quad (26)$$

In the case of small contrast range of intensity resonances, $\mathcal{L}$ depends on u as

$$\mathcal{L} \cong (1/4 + u^2\tau^2)\,(E_0^2)^{-1}, \quad (27)$$

i.e., with a growth in u the level of $\mathcal{L}$ increases. This is consistent with the dependence $\langle\delta\psi^2(\omega=0)\rangle^{1/2}(u)$ in the absence of methane (Fig. 25). Therefore, CH_4 produces no influence on this term at small K.

At high contrast range K resonance structures directed upwards are observed in the dependence of $\mathcal{L}$ on u (if one of the modes coincides with the center of the absorption line). With a growth of K their amplitude rises. Resonance widths in the dependence $\mathcal{L}(u)$ are of the order of $\gamma^{(-)}$ and do not depend on observation frequency ω'.

Exactly this term leads to appearance of resonant structures in the dependence $\langle\delta\psi^2(\omega')\rangle^{1/2}(u)$ directed up at high K.

The second term in (25) represents the autostabilizing effect of methane on $\langle\delta\psi^2(\omega')\rangle^{1/2}$. One can conclude from (25) that the presence of methane imparts an appearance of resonant structures in the dependence $\langle\delta\psi^2(\omega')\rangle^{1/2}(u)$. It should be emphasized that resonance width is defined by the difference $(\gamma^{(-)} - \gamma_0^{(-)})$ of the decay rate of medium polarization and populations of operating levels.

3.3. Experimental Study of Natural Intensity Fluctuations in a Two-Mode Laser

3.3.1. Description of Experimental Set-up. Experimental studies are carried out using an He–Ne laser ($\lambda = 3.39$ μm) generating two linearly and orthogonally polarized modes.

The active laser medium is fixed between phase anisotropic wedges oriented to meet each other. This allows one to perform small adjustments of intermode interactions due to variation of longitudinal shift (δ) at any given intermode interval; simultaneously a smooth tuning of ω_{12} in the interval from 0 to $c / 4L$ is also possible [30].

To vary the intermode interval in a greater range a transverse spatial shift in the laser is used [36]. In the laser resonator a crystal of lithium noibate ($LiNbO_3$) is fixed at the optical path of the beam. Due to birefringence this crystal produces the shift r_0 between mode fields in the transverse direction

$$r_0 = d\Delta h/n, \Delta n = |n_o - n_e|, n \cong n_o \cong n_e,$$

where d is the thickness of $LiNbO_3$ crystal; and $n_{o,e}$ are the refraction indices for ordinary and extraordinary beams.

Variation of laser radiation frequency is provided by scanning resonator length.

Measurements of intensity fluctuations of one of the modes are conducted according to the following technique. Laser radiation (Fig. 6) passes through the polarizer (2) oriented along polarization direction of one of the modes being generated; then this laser radiation is applied to the cooling photodetector (3) with the band 3.5 MHz. The signal from the photodetector output is applied to the low-frequency spectrum analyzer (4) which registers the dependence of intensity fluctuations on frequency $(\langle \delta I_i^{(+)2}(\omega)\rangle^{1/2})$. Laser radiation from another output passes through the polarizer oriented at the angle 45° to the direction of polarization of both modes (5) and then is applied to the wide band photodetector (6). The high-frequency amplifier amplifies the mode beating signal. The signal from this photodetector is used to measure the interval between modes by means of the high-frequency spectrum analyzer (7). With the help of the frequency detector (8) this signal also allows determination of the dependence of intermode beating frequency on detuning. If one knows the dependence of ω_{12} on detuning, mode position in a gain line circuit can be easily obtained.

The dependence of the level of spectral density of natural intensity fluctuations $\langle \delta I_i^{(+)2}(\omega = \omega')\rangle^{1/2}$ on u is measured at fixed frequency ω' in accord with the following technique. The signal from the photodetector output is applied to the spectrum analyzer (4) which detects intensity fluctuations at fixed frequency ω' in the band 3 kHz.

The signal from the spectrum analyzer is applied to the oscillograph (10) which provides the possibility to register the dependence of $\langle \delta I_i^{(+)2}(\omega = \omega')\rangle^{1/2}$ on u. In turn, the signal from the photodiode (3) is applied to the oscillograph (10) which allows one to register the dependence of $\langle I_1^{(+)}\rangle$ on u.

We present the following arguments which ensure that the noise observed is natural [26, 37, 38]: (i) a growth of laser radiation intensity imparts a decrease of natural intensity fluctuations; (ii) the spectrum of natural intensity fluctuations is smooth and constantly decreases. In the case of a symmetric mode position in

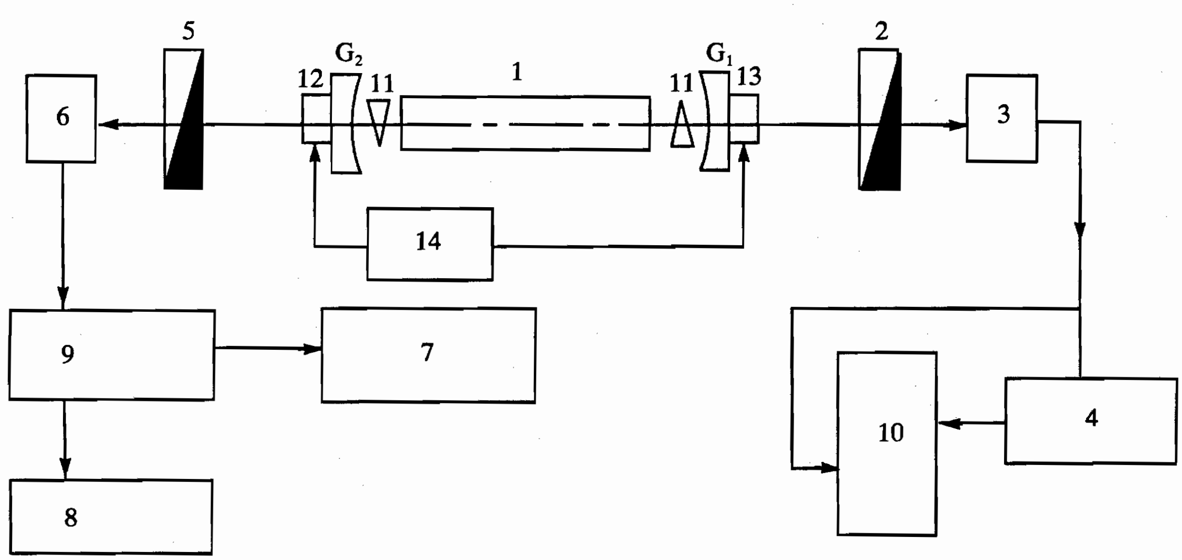

Fig. 6. *The scheme of the experimental set-up: (1) – laser to be investigated, (2, 5) – polarizer, (3, 6) – photodetector, (4) – low-frequency spectrum analyzer, (7) – high-frequency spectrum analyzer, (6) – frequency detector, (9) – high-frequency amplifier, (10) – two beam oscillograph, (11) – phase anisotropic element, (12, 13) – piezocorrector, (14) – generator.*

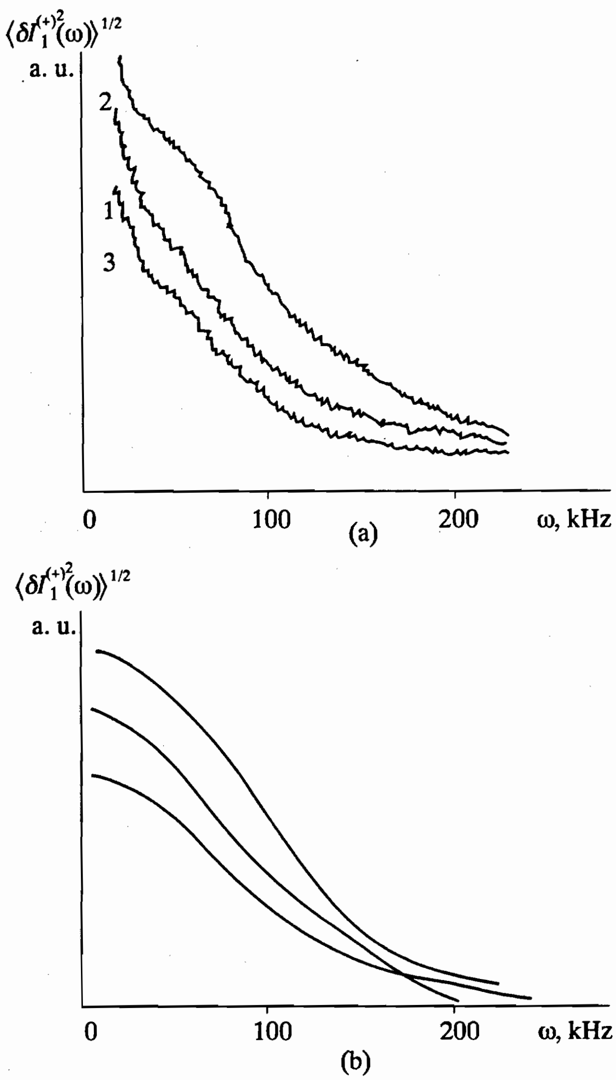

Fig. 7. *Experimental (a) and theoretical (b) dependences of $\langle \delta I_1^{(+)2}(\omega)\rangle^{1/2}$ on u at ω_{12} = 45 MHz, δ = 0. Curve 1 corresponds to u = 0, curve 2 – u = –2 MHz, curve 3 – u = 0.5 MHz.*

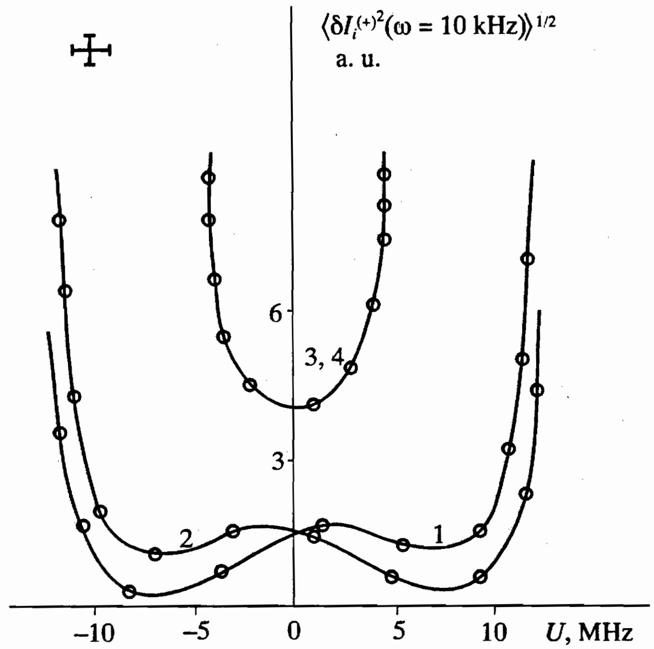

Fig. 8. *Experimental dependence of $\langle \delta I_i^{(+)2}(\omega = 10\ kHz)\rangle^{1/2}$ (i = 1, 2) on u at ω_{12} = 45 MHz, δ = 30°, Δ = 40 MHz (curves 1, 2), at ω_{12} = 8 MHz, δ = 55°, Δ = 17 MHz (curves 3, 4). Curves 1, 3 correspond to the first mode, the curves 2, 4 – to the second mode.*

a gain line circuit it can be adequately approximated by a Lorentzian; (iii) an increase of the degree of intermode coupling at a symmetric mode position reduces the spectral width of natural intensity fluctuations.

3.3.2. Dependence of Natural Intensity Fluctuations Spectra on Mode Position in a Gain Line Circuit. Fig. 7a demonstrates the intensity fluctuation spectrum of one of the modes in the band 0.4 - 400 kHz at various mode positions in a gain line circuit. The low-frequency part of the spectrum (from 0.4 to 4 kHz) is not monotonic. It consists of separate peaks and increases with a growth of the excess. Therefore, technical fluctuations determine this spectrum part. At frequencies higher than 5 kHz the spectrum becomes smooth and constantly decreases. With a growth of the excess at $\omega > 5$ kHz the level of spectral density decreases. For this reason, at higher frequencies ($\omega > 5$ kHz) the fluctuation spectrum is determined by natural radiation fluctuations.

Fig. 7a displays the experimental curves 1, 2, 3 and Fig. 7b shows calculated curves of the natural intensity fluctuations spectrum at various mode positions in a gain line circuit. This figure shows that detuning strongly effects the spectrum profile of natural intensity fluctuations. In the case of a symmetric mode position the natural intensity fluctuation spectrum (curve 1) is adequately approximated by the Lorentz profile (curve 1), but for an arbitrary mode position it obeys a more complicated law (curves 2, 3). Approaching two-mode generation the width of natural intensity fluctuations is dramatically reduced and their level is increased.

In the case of a symmetric mode position in a gain line curve ($u = 0$) the intensity fluctuation spectra of the first and the second modes coincide. At $u \neq 0$ the intensity fluctuation spectra of both modes become essentially different.

Fig. 8 shows experimental dependences of spectral density of mode intensity fluctuations at the frequency $\omega = 10$ kHz and mode detuning from a symmetric position. Curve 1 corresponds to the first mode, and curve 2 – to the second one at $\omega_{12} = 45$ kHz. Curves 2, 3 refer to $\omega_{12} = 8$ kHz and coincide for the first and second modes. A comparison of curves 1 and 2 reveals a mirror image behavior of intensity fluctuations of the first and second modes with respect to the line $u = 0$. At

ω_{12} = 45 kHz the dependences of $\langle \delta I_i^{(+)2}(\omega = 10\ \text{kHz})\rangle$, ($i$ = 1, 2) on detuning have a symmetric character. Their minima do not correspond to a symmetric mode position.

As shown above, this fact is connected with a physical difference in the right and left boundaries of the two-mode generation band of a given mode. With a decrease in ω_{12} the dependences of $\langle \delta I_i^{(+)2}(\omega = 10\ \text{kHz})\rangle$ on u become close to symmetric with respect to the line u = 0. This is associated with the fact that with a decrease in ω_{12} interaction of the modes turns out to be more intensive and mode detunings from a symmetric position become equivalent in the right and left directions.

3.3.3. Dependence of Natural Intensity Fluctuations on the Degree of Intermode Coupling. Variation of the degree of intermode coupling is carried out by means of variation of ω_{12} for a given spectral shift. For a fixed value of ω_{12} the value of S is calculated using (8). The parameters of the active medium are the same as those in the experiment: ku = 170 MHz, $\gamma_b^{(0)}$ = 10 MHz, $\gamma_a^{(0)}$ = 10 MHz, A = 30 MHz/torr, $A_b^{(2)}$ = 5 MHz/torr, $A_a^{(2)}$ = 20 MHz/torr, $p^{(+)}$ = 1.2 torr, η = 1.3. The coefficients $\beta_i^{(+)}$ and $\theta_{ij}^{(+)}$ are calculated according to the limit of a nonuniformly broadened line [27].

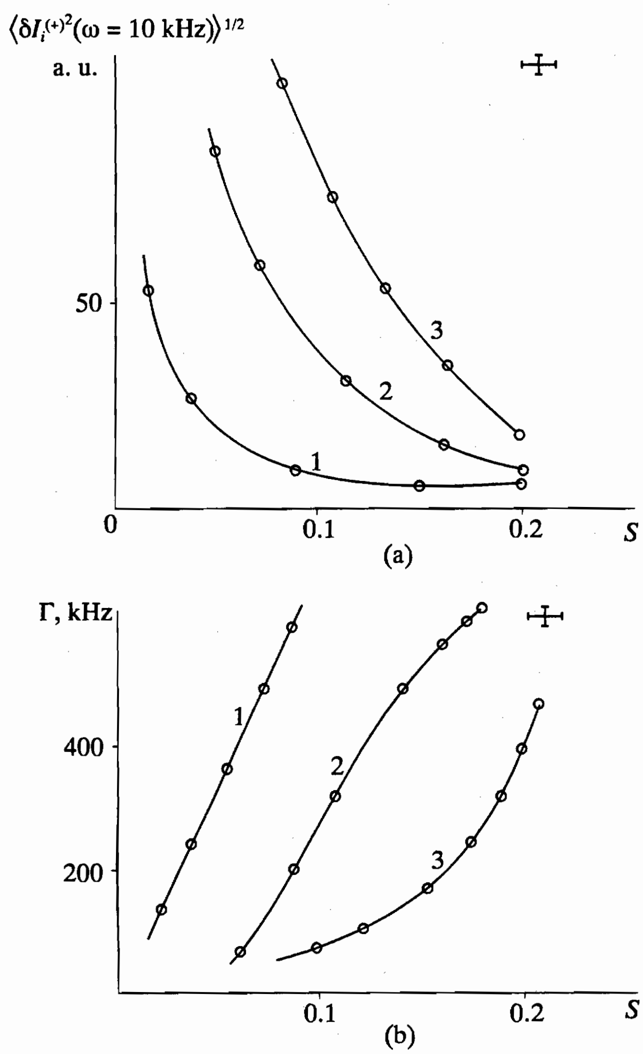

Fig. 9. *Experimental dependence $\Gamma(S)$: (a) $\langle \delta I_i^{(+)2}(\omega = 10\ \text{kHz})\rangle^{1/2}$ on S, (b) at ω_{12} = 25 MHz, δ = 60°, Δ = 40 MHz. Curve 1 corresponds to u = 0, curve 2 – u = 5 MHz, curve 3 – u = 10 MHz.*

Fig. 9 presents experimental dependences of the spectral width of natural intensity fluctuations of the first mode (a) and the spectral density of natural intensity fluctuations (b) (at the frequency ω = 10 kHz) on the degree of intermode coupling at various values of u.

With an increase in S (a decrease of intermode coupling) $\langle \delta I_1^{(+)2}(\omega = 10\ \text{kHz})\rangle^{1/2}$ falls. With a growth in u a slope of the dependence $\langle \delta I_1^{(+)2}(\omega = 10\ \text{kHz})\rangle^{1/2}$ on S becomes higher. As one can deduce from (9) this appears due to changing of mode intensities at a growth of intermode coupling.

From a qualitative viewpoint this can be interpreted in the following way. The degree of intermode coupling weakly depends on mode position in a gain line circuit. In fact, it can be taken to be constant. In the case of strong coupling mode intensity linearly depends on detuning. Approaching the boundaries of two-mode generation its noise intensity grows. Due to intermode coupling the noise in the second mode also increases. As the given mode continues to move away from a symmetric position natural intensity fluctuations of this mode becomes greater due to intensity decline. Therefore, as this proceeds natural intensity fluctuations increase because of intermode coupling. Consequently, at $u \neq 0$ a growth of intermode coupling leads to faster growth of natural intensity fluctuations than I / S due to intensity redistribution between modes (as found in [26]).

With a growth in S the spectrum width of natural intensity fluctuations rises at any mode position in a gain line circuit (Fig. 9). With an increase in mode detuning from a symmetric position the dependence of Γ on S grows faster.

As mode detuning from a symmetric position grows the degree of intermode coupling produces stronger influence on the level of $\langle \delta I_1^{(+)2}(\omega = 0)\rangle^{1/2}$ and the width of spectral density of natural intensity fluctuations of a mode (Γ_1).

3.3.4. Effect of Spatial Separation of Mode Fields on Natural Intensity Fluctuations. It is shown above that variation of the degree of intermode coupling

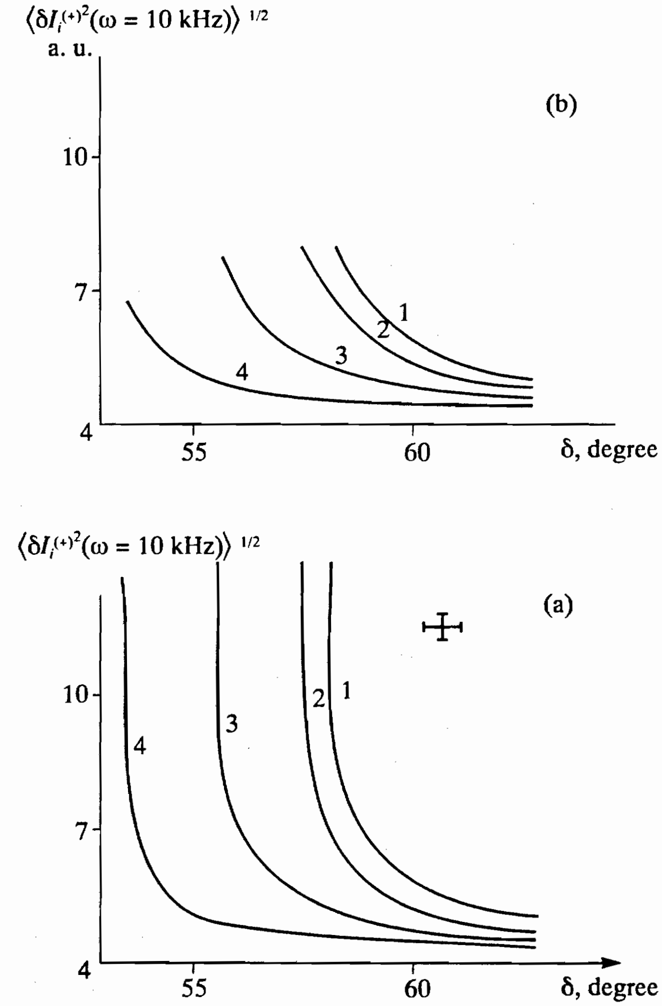

Fig. 10. *Experimental (a) and calculated (b) dependence of* $\langle \delta I_i^{(+)2}(\omega = 10\ kHz)\rangle^{1/2}$ *on* δ *at: (1) –* $\omega_{12} = 2$ *MHz, (2) –* $\omega_{12} = 4$ *MHz, (3) –* $\omega_{12} = 6$ *MHz, (4) –* $\omega_{12} = 10$ *MHz.*

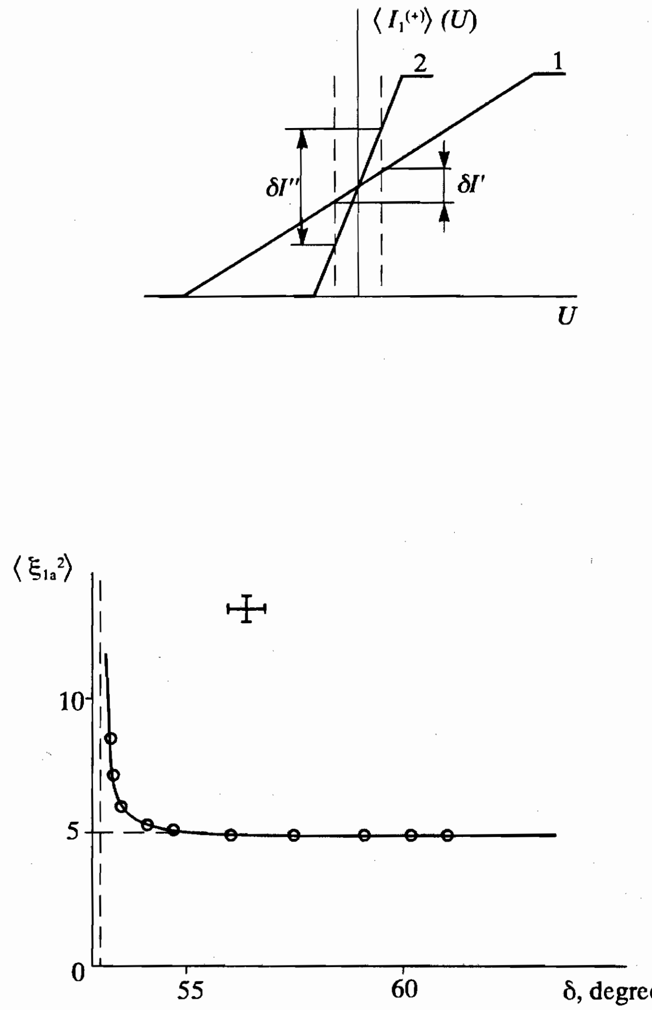

Fig. 11. *Experimental dependence of* $\langle \xi_{1a}^2 \rangle$ *on* δ *at* ω_{12} = *10 MHz (a); dependence of* $\langle I_1^{(+)} \rangle$ *on u at* $\delta = \pi/2$ *(curve 1) and at* $\delta = \delta_{cr}$ *(curve 2) (b).*

greatly affects natural intensity fluctuations. Longitudinal and transverse shifts of mode fields ensure the most profound change in intermode coupling. But spatial separation of mode fields affects natural intensity fluctuations not only due to change in S but also because of change in correlation of noise sources in the modes. For example, if noise sources in the modes are correlated at their spatial coincidence, then it is clear that for complete transverse mode separation correlation should disappear. For this reason the study of the effect of spatial mode separation on natural intensity fluctuations is contained in the present paper in a special section.

To avoid accessory phenomena associated with the influence of detuning on natural intensity fluctuations an analysis of the effect of spatial separation of mode fields on natural intensity fluctuations is performed for the case of a symmetric mode position in a gain line circuit.

Fig. 10a, b demonstrates calculated and experimental dependences of $\langle \delta I_1^{(+)2}(\omega' = 10\ kHz)\rangle^{1/2}$ on longitudinal shift (the value of ω' is ≅10 kHz, because at these frequencies technical fluctuations in fact produce no influence on the spectrum of natural intensity fluctuations). At the same time this frequency can be assumed to be small in comparison with spectrum width. The parameters of the active medium used in calculations are the same as in Fig. 4. Fig. 10 shows that with a decrease in δ from π/2 to 0 the level of $\langle \delta I_1^{(+)2}(\omega' = 10\ kHz)\rangle^{1/2}$ increases more than 300 times. In the range $\delta = \pi/2$, the behavior of the experimental and theoretical curves is similar. With a decrease in δ a small growth in natural intensity fluctuations is observed. At $\delta \sim \delta_{cr}$ (δ_{cr} is the minimal spatial shift at which stable two-mode generation can be obtained) the experimental dependence of $\langle \delta I_1^{(+)2}(\omega = \omega')\rangle^{1/2}$ on δ increases is essence faster than the calculated one. Therefore, one can suggest that in the range $\delta \sim \delta_{cr}$ an additional mechanism which controls the dependence of intensity fluctuations on δ exists. This mechanism appears to be even

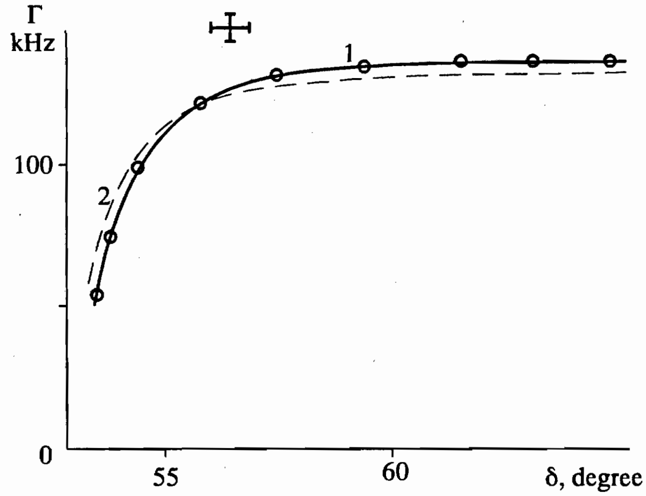

Fig. 12. *Experimental (curve 1) and calculated (curve 2) dependences $\Gamma(\delta)$ at ω_{12} = 8 MHz.*

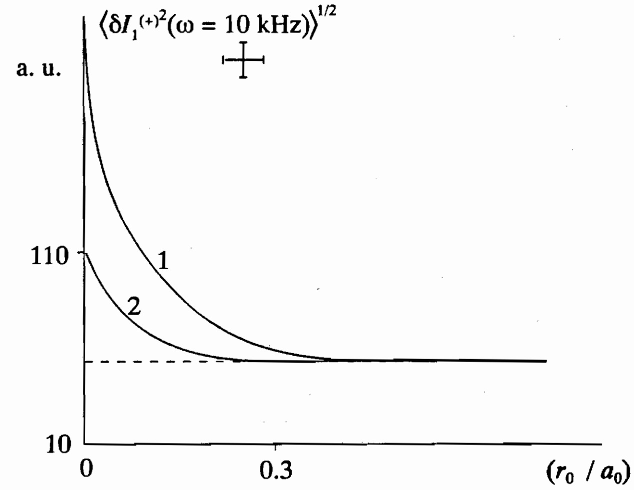

Fig. 13. *Experimental (curve 1) and calculated (curve 2) dependences of $\langle \delta I_1^{(+)2}(\omega = 10\ kHz)\rangle^{1/2}$ on r_0 / a_0 at ω_{12} = 45 MHz.*

stronger than the dependence of natural intensity fluctuations on the degree of intermode coupling.

To perform a more detailed investigation of intensity fluctuations in the range $\delta \cong \delta_{cr}$ the dependence of $\langle \xi_{1a}^2 \rangle$ on δ is plotted using experimental results by means of (9) (see Fig. 11a). In the range $\delta > \delta_{cr}$ the value of $\langle \xi_{1a}^2 \rangle$ is constant. At $\delta \rightarrow \delta_{cr}$ the value of $\langle \xi_{1a}^2 \rangle$ grows sharply. Therefore, in the range $\delta \rightarrow \delta_{cr}$, a dependence of $\langle \xi_{1a}^2 \rangle$ occurs on spatial shift δ, which is not included in our mathematical model. It seems possible that the existence of such a dependence near $\delta \rightarrow \delta_{cr}$ is associated with an increase in the effect of frequency fluctuations on intensity fluctuations. Due to the approach of the two-mode generation band to zero at $\delta \rightarrow \delta_{cr}$ even small variations of mode frequency cause considerable changes in intensity.

Let us clarify this circumstance by a simple example. In Fig. 11b the dependence of $\langle I_1^{(+)} \rangle$ on u corresponding to the case of the weakest intermode coupling ($\delta \cong \pi / 2$) (curve 1) and that of strong intermode coupling ($\delta \cong \delta_{cr}$)(curve 2) is presented. Let the level of frequency noise in both cases be the same and determined by the deviation T. Then frequency deviation in T changes the intensity of the values $\delta I'$ in the first case, but in the second case intensity deviation changes the intensity of $\delta I''$. Because the steepness of the dependence $\langle I_1^{(+)} \rangle$ on u at $\delta \cong \delta_{cr}$ is much higher than at $\delta \cong \pi / 2$, one finds $\delta I'' \gg \delta I'$. Thus, in the range $\delta \sim \delta_{cr}$, the influence of frequency noise on natural intensity fluctuations may be observed (this influence should be stronger at $\delta \sim \delta_{cr}$).

Fig. 12 demonstrates calculated (1) and experimental (2) dependences of the spectrum width of natural intensity fluctuations on the spatial shift δ. It can be seen from this figure that with a decrease in δ from $\pi / 2$ to δ_{cr} the spectrum width falls. It should be added that within experimental error the calculated and experimental dependences have similar behavior.

For the case of a symmetric mode position in a gain line circuit the spectrum width can be approximated by the expression [26]

$$\Gamma = \alpha_1^{(+)} (\beta_1^{(+)} - \theta_{12}^{(+)}) / (\beta_2^{(+)} + \theta_{12}^{(+)}).$$

It is easy to notice that change in δ exclusively affects the spectrum width of natural intensity fluctuations by a change in the degree of intermode coupling. There is no evidence here regarding the influence of the dependence of $\langle \xi_{1a}^2 \rangle$ on δ.

In practice the problem of particular reduction of natural intensity fluctuations for two-mode generation is often of special importance. If the two-mode generation band is broad, intermode coupling becomes a factor which substantially affects the level of spectral density of natural intensity fluctuations. In transverse mode separation intermode coupling can be especially effectively controlled. With a growth in r_0 (a spatial shift between the modes in the transverse direction rises) the factor S approaches 1 and intermode coupling can be arbitrarily weak. In turn, this ensures that the spectral density of natural intensity fluctuations declines and in the limiting case it should approach the spectral density of intensity fluctuations for one-mode generation.

Fig. 13 displays experimental (curve 1) and calculated (curve 2) dependences of $\langle \delta I_1^{(+)2} (\omega = 10\ \text{kHz})\rangle^{1/2}$ on the transverse shift r_0 / a_0 (a_0 is the mode dimension). Experimental studies aimed to investigate the effect of mode field separation on natural intensity fluctuations are performed for a symmetric mode position in a gain line circuit. Electric discharge preserves the output power of laser radiation constant as r_0 / a_0 varies. The dependence of noise sources on spatial shift is unknown. Therefore, in calculation $\langle \xi_{1a}^2 \rangle (1 - K_{12}^R)$ are

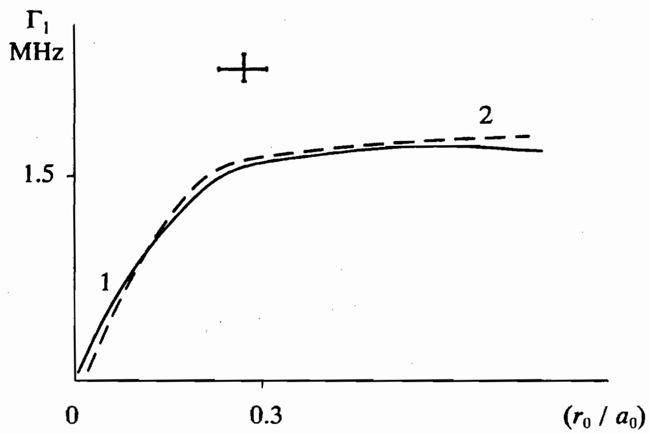

Fig. 14. *Experimental (curve 1) and calculated (curve 2) dependences of Γ on r_0 / a_0 at ω_{12} = 10 MHz.*

constant. This probably explains some quantitative distinction in the behavior of the experimental and calculated curves. In the calculations the coefficients $\alpha_i^{(+)}$, $\beta_i^{(+)}$ and $\theta_{ij}^{(+)}$ with account to transverse mode field separation are taken from [36].

The figure shows that with a growth in transverse shift the level of natural intensity fluctuations drops and at $(r_0 / a_0) \cong 0.3$ it is practically equal to that for one-mode generation.

Good agreement between the calculated (curve 2) and experimental (curve 1) dependences of $\langle \delta I_1^{(+)2} (\omega = 10 \text{ kHz}) \rangle^{1/2}$ on r_0 / a_0 argues that the basic mechanism which affects natural intensity fluctuations at a transverse shift is a variation of the degree of intermode coupling.

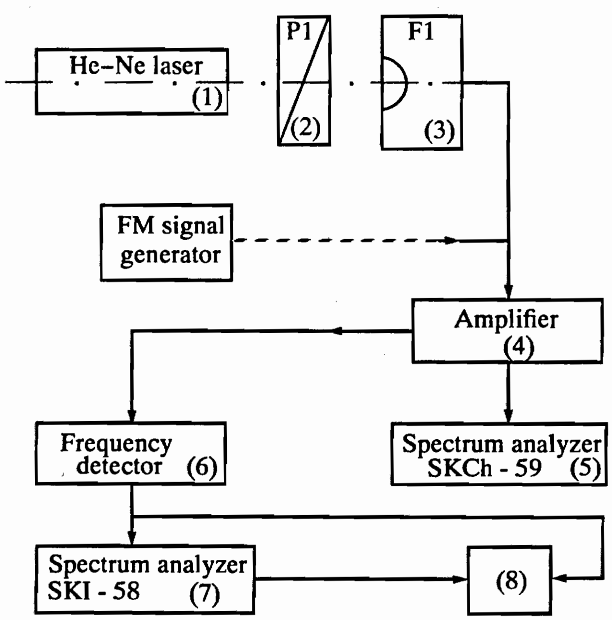

Fig. 15. *The scheme of the experimental set-up: (1) – laser to be investigated, (2) – polarizer, (3) – photodetector, (4) – amplifier, (5) – high frequency spectrum analyzer, (6) – frequency detector, (7) – low-frequency spectrum analyzer, (8) – two beam oscillograph.*

At the same time transverse shift produces no noticeable impact on the level of natural intensity fluctuations by changing the correlation coefficients of noise sources in the modes.

Fig. 14 shows the experimental (curve 1) and calculated (curve 2) dependences of spectrum width of natural intensity fluctuations on transverse shift. Measurements of Γ are carried out at small values of the excess $\eta \cong 1.1$. It can be seen from the figure that with a growth in r_0 / a_0 spectrum width natural intensity fluctuations increase and approach the width for one- mode generation. The calculated and experimental dependences agree well, which confirms analytical dependences for natural intensity fluctuations already obtained.

3.4. Experimental Study of Natural Fluctuations of Intermode Beating Frequency

Measurements of natural fluctuations of intermode beating frequency are conducted according to the following technique (Fig. 15). Laser output radiation is passed through the polarizer (P1) oriented at the angle 45° to the directions of mode polarization and then illuminates the photodetector (PD1). The signal from the frequency photodetector is applied to the broad-band amplifier (4); then part of the signal is applied to the high- frequency spectrum analyzer (5), and another applied to the frequency detector (6). The signal from the frequency detector is applied to the low-frequency spectrum analyzer (7) which allows one to register the spectrum of intermode beating frequency fluctuations.

The chosen techniques provide an opportunity to study intermode beating frequency fluctuations in the band 0 - 200 kHz.

3.4.1. Discussion. Fig. 16 displays experimental dependence of the spectral density of intermode beating frequency fluctuations on the (ω) for a symmetric mode position in a gain line circuit. Our equipment allows one to perform spectrum measurements only at frequencies less than 200 kHz. In the band 0.04 - 4 kHz the fluctuation spectrum is not monotonic and consists of separate peaks due to purely technical reasons. With a growth of output radiation intensity the level of the spectral density of intermode beating frequency fluctuations grows. Consequently, the low-frequency end of the spectrum of intermode beating frequency fluctuations is controlled by technical fluctuations.

In the interval 4 - 200 kHz the fluctuation spectrum does not actually depend on the frequency ω and with a growth in intensity decreases as $1 / \langle I^{(+)} \rangle$ (Fig. 17). The level of receiver noises measured in accordance with the technique described above is much less than intermode beating frequency fluctuations. Therefore, one

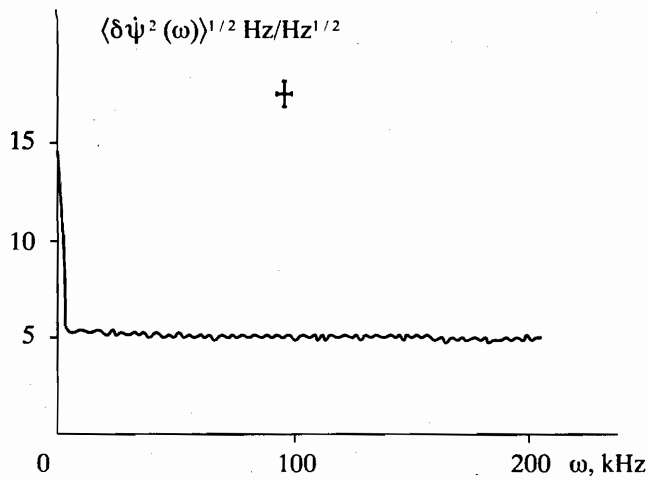

Fig. 16. *Experimental dependence of* $\langle \delta\dot{\psi}^2(\omega) \rangle^{1/2}$ *at* ω_{12} *= 39 MHz,* δ *= 60°.*

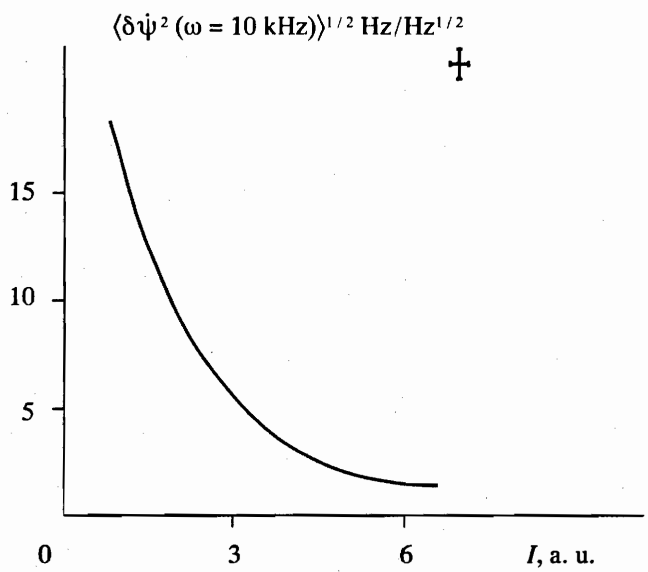

Fig. 17. *Experimental dependence of* $\langle \delta\dot{\psi}^2(\omega = 10\ kHz) \rangle^{1/2}$ *on I at* ω_{12} *= 39 MHz,* δ *= 60°.*

can conclude that the high-frequency end of the spectrum (ω > 4 kHz) is controlled by natural fluctuations.

One can conclude from (12) that the spectrum of natural fluctuations of intermode beating frequency in the band 0.4 - 200 kHz can be in fact constant, if it is specified by purely frequency noises or amplitude noises with width 200 kHz (see Fig. 18). Therefore, it is rather difficult to obtain the spectrum profile and consequently to distinguish the contribution of natural intensity fluctuations and frequency noises to $\langle \delta\dot{\psi}^2(\omega) \rangle^{1/2}$.

As stated above, increasing the degree of intermode coupling one can reduce the spectrum width of natural intensity fluctuations up to a band less than 200 kHz. In this case the level of spectral density of natural fluctuations grows. In the experiments variation of the intermode interval actually controls the degree of intermode coupling. Experimental variation of ω_{12} in a wide range from 0.5 to 40 MHz does not effect the spectrum profile. In this case the spectrum width of natural intensity fluctuations is reduced to 40 - 50 kHz. Experimental dependence of $\langle \delta\dot{\psi}^2(\omega) \rangle^{1/2}$ on ω_{12} is plotted in Fig. 2 (curve 4). This figure demonstrates that change in ω_{12} does not affect the level of spectral density of natural frequency fluctuations.

In accordance with (12) one finds that this is possible if intensity fluctuations do not impact natural fluctuations of intermode beating frequency.

For changing ω_{12} influence of the third term in (12) on $\langle \delta\dot{\psi}^2(\omega) \rangle^{1/2}$ is to be observed (only if the intensity and frequency fluctuation sources are correlated in any way). Thereby, the sources ξ_{ia} and ξ_{jr} are not correlated, i.e.

$$\langle \xi_{ia}(t)\, \xi_{ir}(t') \rangle = 0.$$

Therefore, natural fluctuations of intermode beating frequency are dictated by purely frequency noises. Their spectrum profile appears constant.

It is easy to conclude from (12) that the level of natural fluctuations of intermode beating frequency strongly depends on correlation between frequency noise sources.

To determine correlation between frequency noise sources the dependence of $\langle \delta\dot{\psi}^2(\omega) \rangle^{1/2}$ on δ is to be studied. Fig. 3 (curve 3) displays

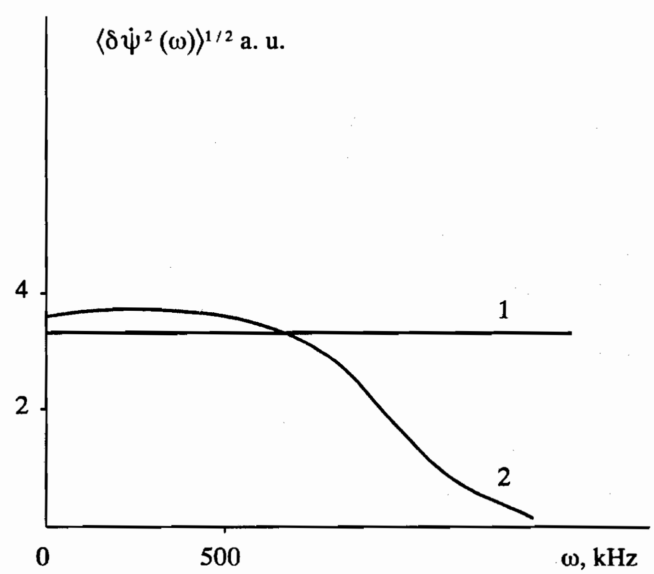

Fig. 18. *Calculated dependence of the spectrum: (1) for purely frequency noises, (2) for the case of dominance of natural intensity fluctuation noises at* ω_{12} *= 45 MHz,* δ *= 30°.*

$\langle \delta\dot{\psi}^2(\omega = 10\text{ kHz})\rangle^{1/2}$ as a function of δ. In practice, variation of δ in the interval from $\pi/2$ to δ_{cr} (δ_{cr} is the minimal spatial shift at which two-mode generation occurs) does not influence the level of $\langle \delta\dot{\psi}^2(\omega = 10\text{ kHz})\rangle^{1/2}$. Consequently, frequency noise sources are weakly correlated, i.e.

$$\langle \xi_{1r}(t)\xi_{2r}(t')\rangle \sim 0.$$

Correlation between frequency noise sources is investigated for the case of small excess in the laser (i.e., $\eta \sim 1.05$ - 1.2). As shown above, for the case of small excess an analogous effect takes place for natural intensity fluctuation sources.

In the case of a symmetric mode position in a gain line circuit one can simplify (12) for $\langle \delta\dot{\psi}^2(\omega)\rangle$ by using the analysis described here

$$\langle \delta\dot{\psi}^2(\omega)\rangle^{1/2} = \omega_0\langle\xi_r^2\rangle\sqrt{2}/\langle I^{(+)}\rangle, \quad (28)$$

where in accordance with a symmetric mode position one has $\langle\xi_{1r}^2\rangle = \langle\xi_{2r}^2\rangle = \langle\xi_r^2\rangle$. This implies that the level of intermode beating frequency fluctuations is $\sqrt{2}$ times greater than the level of frequency fluctuations in a one-mode laser with the same output power as that of one of the modes for two-mode generation. From the physical viewpoint this means that the frequencies of the modes being generated fluctuate independently. Therefore, the level of intermode beating frequency fluctuations is equal to the sum of frequency fluctuations for each individual mode.

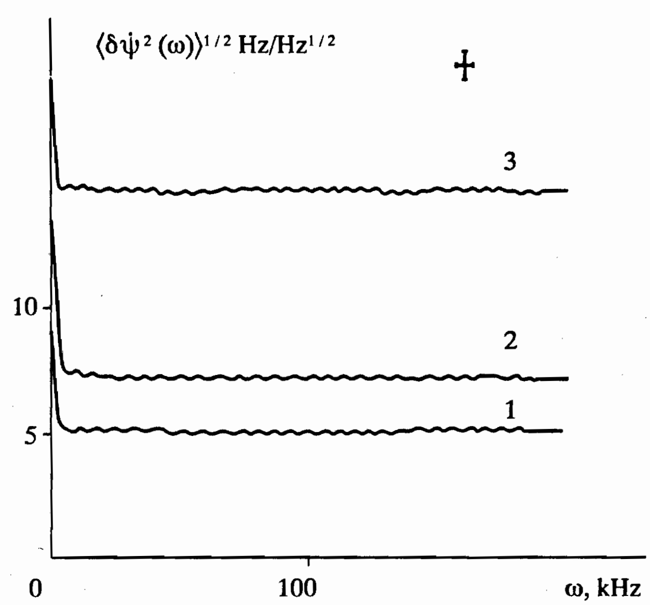

Fig. 19. *Experimental dependence of $\langle \delta\dot{\psi}^2(\omega)\rangle^{1/2}$ at various mode positions in a gain line curve, if $\omega_{12} = 10$ MHz, $\delta = 60°$. (1) – $u = 0$ MHz, (2) – $u = 5$ MHz, (3) – $u = 7$ MHz.*

<u>3.4.2. Effect of Mode Detuning from a Symmetric Position on Intermode Beating Frequency Fluctuations.</u> Fig. 19 presents the spectrum profile of $\langle \delta\dot{\psi}^2(\omega)\rangle^{1/2}$ measured experimentally at various mode positions in a gain line circuit. Curve 1 corresponds to $u = 0$, 2 – to $u = 5$ MHz and curve 3 to $u = -7$ MHz. One can see from this figure that mode detuning from a symmetric position does not impact the spectrum profile. This spectrum can be adequately approximated by a constant for any mode position. As already established, this becomes possible, if the contribution of sources of purely frequency fluctuations to $\langle \delta\dot{\psi}^2(\omega)\rangle^{1/2}$ is much higher than that of intensity fluctuations. Consequently, for $\langle \delta\dot{\psi}^2(\omega)\rangle^{1/2}$ natural intensity fluctuations in (12) can be ignored at an arbitrary mode position. Then the expression for $\langle \delta\dot{\psi}^2(\omega)\rangle^{1/2}$ takes the form

$$\langle \dot{\psi}^2(\omega)\rangle^{1/2} = \omega_0\langle\xi_r^2\rangle^{1/2}(\langle I_1^{(+)}\rangle + \langle I_2^{(+)}\rangle)/\langle I_1^{(+)}\rangle\langle I_2^{(+)}\rangle. \quad (29)$$

Physically this means that intensity fluctuations produce no influence on frequency fluctuations for two-mode generation.

Fig. 5 shows $\langle \delta\dot{\psi}^2(\omega')\rangle^{1/2}$ as a function of u. One can see that the dependences are adequately approximated by parabolas (their vertices coincide with a symmetric mode position). At the boundaries of two-mode generation the level of $\langle \delta\dot{\psi}^2(\omega')\rangle^{1/2}$ rises. The minimum of intermode beating frequency fluctuations corresponds to a symmetric mode position.

As the mode approaches u_0 at fixed frequency the spectral density of intermode beating frequency fluctuations can be expressed as follows

$$\langle \delta\dot{\psi}^2(\omega')\rangle_{u\to u_0} = \omega_0^2\langle\xi_r^2\rangle\frac{1}{\langle I_1^{(+)}\rangle}, \quad \langle I_1^{(+)}\rangle \to 0.$$

If the mode approaches $(-u_0)$ one gets

$$\langle \delta\dot{\psi}^2(\omega')\rangle_{u\to -u_0} = \omega_0^2\langle\xi_r^2\rangle\frac{1}{I_2^{(+)}}, \quad \langle I_2^{(+)}\rangle \to 0.$$

Therefore, approaching two-mode generation the level of fluctuations increases, and the dependence of $\langle \delta\dot{\psi}^2(\omega')\rangle^{1/2}$ on u is symmetric with respect to the line $u = 0$. In accordance with the approximation of (18) the level of $\langle \delta\dot{\psi}^2(\omega')\rangle^{1/2}$ at mode detuning from a symmetric position increases as

$$\left(1 - \frac{4B^2}{I_{\text{sum}}^2}u^2\right)^{-1}.$$

Qualitatively this can be interpreted in the following manner. Near the boundary of two-mode generation one of the modes approaches a threshold, and thereby its fluctuations considerably grow. In turn, as stated

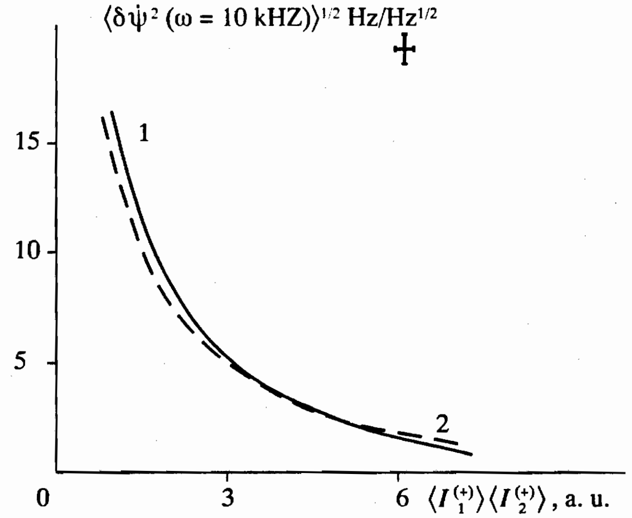

Fig. 20. *Experimental (curve 1) and theoretical (curve 2) dependences of* $\langle \delta\dot{\psi}^2(\omega = 10\,kHz)\rangle^{1/2}$ *on* $\langle I_1^{(+)}\rangle\langle I_2^{(+)}\rangle$ *at* $\omega_{12} = 20\,MHz$ *and* $\delta = 50°$.

above, $\langle \delta\dot{\psi}^2(\omega')\rangle^{1/2}$ depends on the sum of frequency fluctuations of separate modes. Consequently, the level of $\langle \delta\dot{\psi}^2(\omega')\rangle^{1/2}$ near the boundary of two-mode generation must increase.

Experimental (curve 1) and calculated (curve 2) dependences of $\langle \delta\dot{\psi}^2(\omega')\rangle^{1/2}$ on $\langle I_1^{(+)}\rangle\langle I_2^{(+)}\rangle$ are plotted in Fig. 20. One can notice that the larger $\langle I_1^{(+)}\rangle\langle I_2^{(+)}\rangle$ is, the lower the level of $\langle \delta\dot{\psi}^2(\omega')\rangle^{1/2}$ must be. A variation of $\langle I_1^{(+)}\rangle\langle I_2^{(+)}\rangle$ is performed by detuning.

The calculated curve adequately approximates the experimental curve by means of (28). At mode detuning from a symmetric position the level of total intensity remains close to a constant, i.e. $\langle I_1^{(+)}\rangle + \langle I_2^{(+)}\rangle =$ const. Therefore, a growth in $\langle I_1^{(+)}\rangle\langle I_2^{(+)}\rangle$ for scanning of the modes over a gain line circuit results in a decrease in $\langle \delta\dot{\psi}^2(\omega')\rangle^{1/2}$. Thus, as follows from (29) a variation of the level of $\langle \delta\dot{\psi}^2(\omega')\rangle^{1/2}$ vs. u grows due to redistribution of power between the modes.

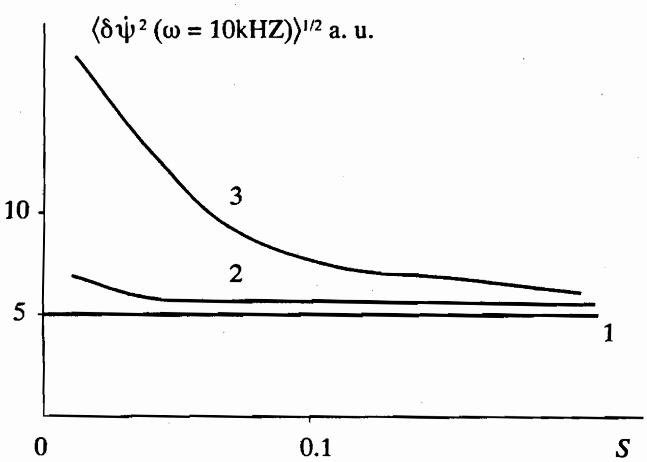

Fig. 21. *Calculated dependence of* $\langle \delta\dot{\psi}^2(\omega = 10\,kHz)\rangle^{1/2}$ *on S at u = 0 MHz (1), u = 3 MHz (2), u = 15 MHz (3),* $\omega_{12} = 45\,MHz$.

3.4.3. Effect of Degree of Intermode Coupling on Natural Fluctuations of Intermode Beating Frequency. The degree of intermode coupling turns out to be an important parameter which specifies laser operating regimes. It controls the two-mode generation band, the dependence of output power on detuning etc. In addition, S essentially affects intensity fluctuations of a single mode. Meanwhile, in the case of a symmetric mode position in a gain line circuit, the magnitude $\langle \delta\dot{\psi}^2(\omega')\rangle^{1/2}$ does not depend on S. But one can see in Fig. 21 that curves 1 and 2 coincide only if $u = 0$. At $u \neq 0$ curve 1 (for this curve the degree of intermode coupling is higher) substantially differs from curve 2. Therefore, a mechanism which imparts an effect of S on the level $\langle \delta\dot{\psi}^2(\omega')\rangle^{1/2}$ exists.

Fig. 21 presents the calculated curve of $\langle \delta\dot{\psi}^2(\omega')\rangle^{1/2}$ vs. S at various mode positions in a gain line circuit. One can see that S essentially influences $\langle \delta\dot{\psi}^2(\omega')\rangle^{1/2}$ only at mode detuning from a symmetric position. With a decrease in S (the degree of intermode coupling increases) the level of $\langle \delta\dot{\psi}^2(\omega')\rangle^{1/2}$ grows. Moreover, the larger $|u|$ is, the larger the rate of $\frac{\partial}{\partial S}\langle \delta\dot{\psi}^2(\omega')\rangle^{1/2}$ on S must be.

This is connected with the increase of intermode coupling reduces the two-mode generation band. For a given mode position in a gain line circuit, a reduction of the two-mode generation band results in intensity redistribution between the modes. Finally, the value of $\langle I_1^{(+)}\rangle\langle I_2^{(+)}\rangle$ falls, and $\langle I_1^{(+)}\rangle\langle I_2^{(+)}\rangle$ remains constant. In accordance with (28) this ensures a growth in $\langle \delta\dot{\psi}^2(\omega')\rangle^{1/2}$.

3.5. Non-Linear Resonances of Natural Intensity Fluctuations in a Two-Mode He–Ne / CH_4 Laser

A He–Ne / CH_4 laser which generates two linearly and orthogonally polarized modes is employed in the experiments (note the interaction between the modes can be smoothly varied).

The methanol cell connected with a vacuum unit is fixed inside the resonator, which allows one to perform measurements at various pressures of $CH_4(p^{(-)})$. The experimental set-up provides an opportunity to investigate the spectrum of natural intensity fluctuations of one of the modes at an arbitrary mode position in a gain line circuit.

The studies show that starting with the frequencies $\omega > 10$ kHz the spectrum of natural intensity fluctua-

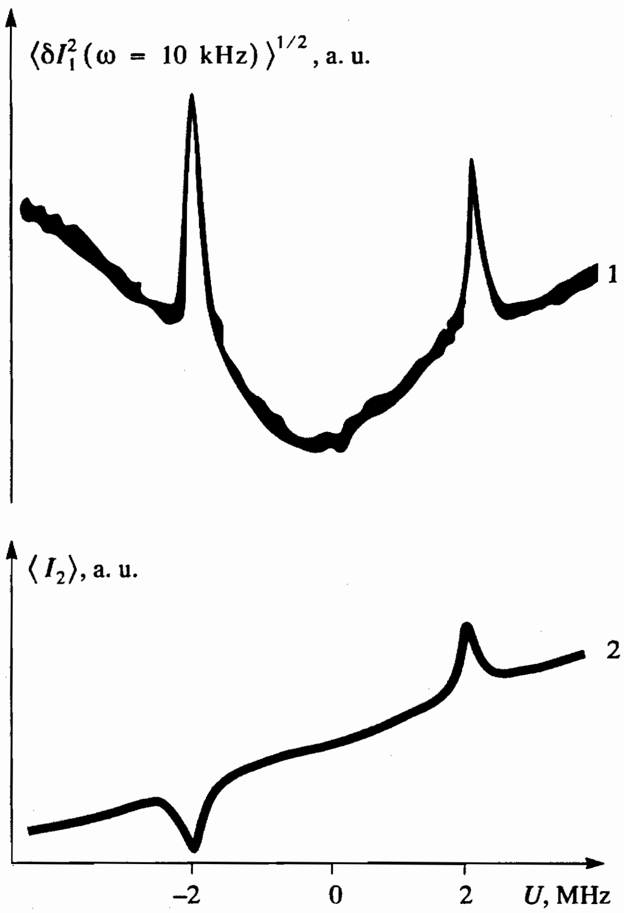

Fig. 22a. *Experimental dependence of* $\langle \delta I_1^2 (\omega = 10\ kHz)\rangle^{1/2}$ *on u (curve 1) and* $\langle I_2 \rangle$ *on u (curve 2) at* $\omega_{12} = 4$ *MHz and* $\delta = 90°$.

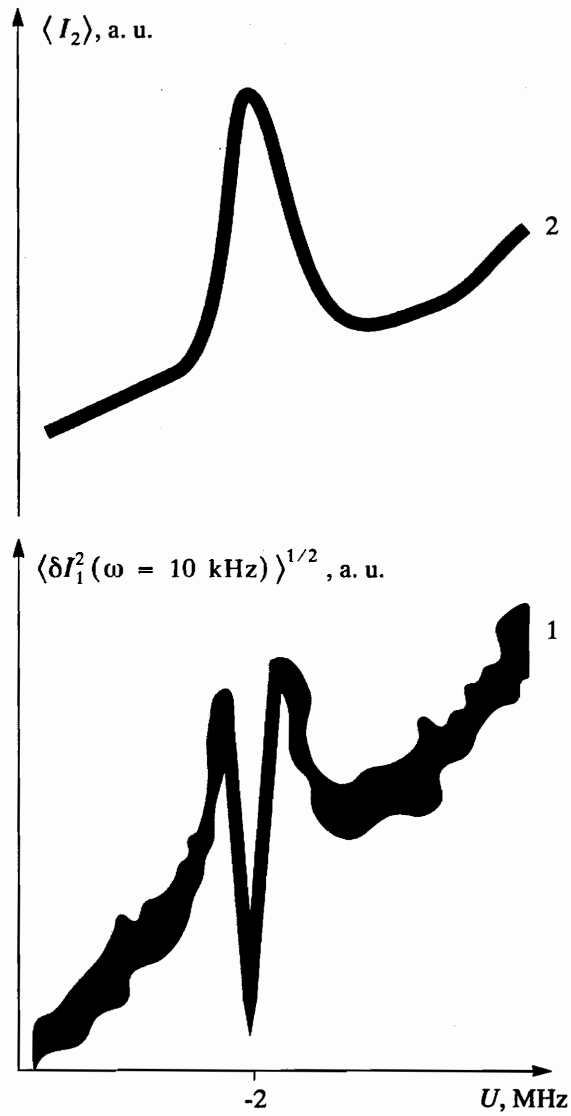

Fig. 22b. *Experimental dependence of* $\langle \delta I_1^2 (\omega = 10\ kHz)\rangle^{1/2}$ *on u (curve 1) and* $\langle I_2 \rangle$ *on u (curve 2) at* $\omega_{12} = 4$ *MHz and* $\delta = 85°$.

tions of the modes becomes smooth and depending on the operating laser parameter constantly decreases with the indicative width 50 - 400 kHz. In turn, in the case of a symmetric mode position the dependence on radiation intensity obeys the law $1 / \langle I_1^{(+)} \rangle$. Therefore, with $\omega > 10$ kHz the spectrum of $\langle \delta I_i^2(\omega) \rangle^{1/2}$ is determined by natural noises.

Fig. 22a displays experimental dependences of $\langle \delta I_1^2(\omega = 10\ \text{kHz}) \rangle^{1/2}$ on u (curve 1) and $\langle I_2 \rangle$ on u (curve 2), if intermode interval is equal to $\omega_{12} = 4$ MHz and the contrast range of intensity resonances is 10%. The resonances directed up with width of the order of the intensity resonance width are observed in the dependence of $\langle \delta I_1^2(\omega = 10\ \text{kHz}) \rangle^{1/2}$ on u, if each mode coincides with the center of the absorption line.

With a growth in intermode coupling, the intensity resonance contrast range is increased, and noise resonance is transformed in such a manner that resonance structure with the width of 1/3 that of intensity resonance appears (Fig. 22b). The profile of resonance structure which occurs is similar to the first derivative of intensity resonance, i.e. the maxima of $\langle \delta I_1^2(\omega = 10\ \text{kHz}) \rangle^{1/2}$ correspond to the center of the intensity resonance slopes; in turn, the minima correspond to the vertex.

This fact is probably associated with the fact that those frequency fluctuations produce stronger effect on natural intensity fluctuations. It is already established that, with a growth in the degree of intermode coupling, the two-mode generation band reduces and frequency fluctuations heavily influence natural intensity fluctuations.

With a growth in intermode coupling, the steepness of intensity resonance becomes higher, which results in additional growth of influence of frequency fluctuations on natural intensity fluctuations. For frequency noise, intensity resonance turns out to be a discrimina-

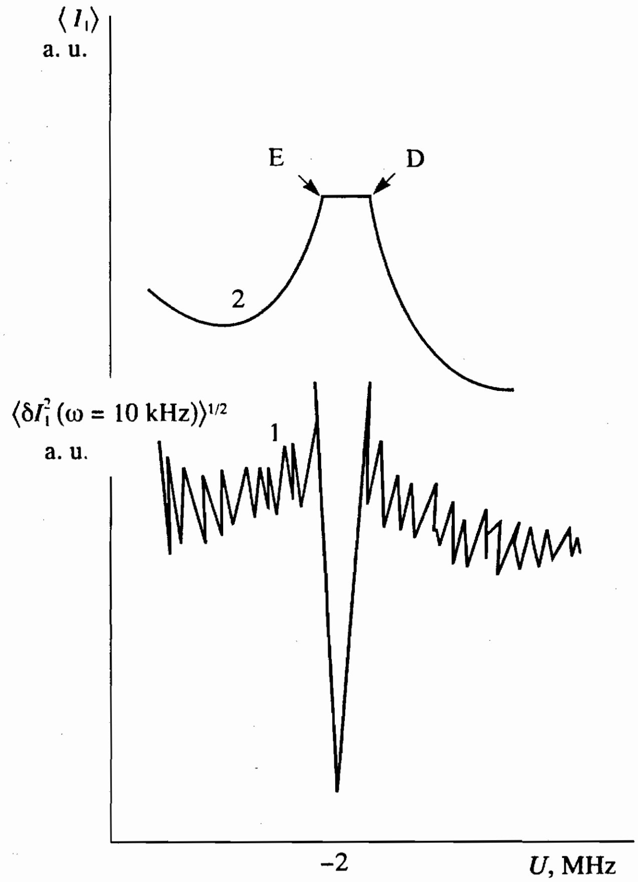

Fig. 22c. *Experimental dependence of* $\langle \delta I_1^2\ (\omega = 10\ kHz)\rangle^{1/2}$ *on u (curve 1) and* $\langle I_1 \rangle$ *on u (curve 2) at* ω_{12} *= 4 MHz and* $\delta = 75°$.

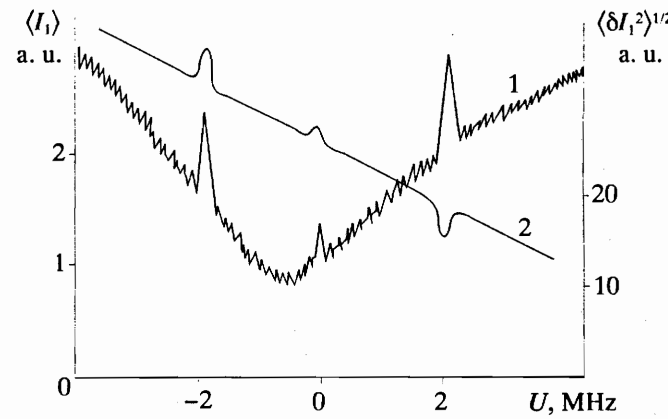

Fig. 23. *Experimental dependence of* $\langle \delta I_1^2\ (\omega = 10\ kHz)\rangle^{1/2}$ *on u (curve 1) and* $\langle I_1 \rangle$ *on u (curve 2) at various quality factors of the modes* ω_{12} *= 4 MHz and* $\delta = 90°$.

tor, therefore the maximum value of natural intensity fluctuations corresponds to the centers of resonance intensity slopes, where the steepness of the dependence of $\langle I_1 \rangle$ on u is a minimum. As a result, the width of the resonance structure formed should be 1/3 of the intensity resonance.

The situation where at the vertex of intensity resonance two-mode generation switches to one-mode is also possible (Fig. 22c). In this case, the structures that are much more narrow than the width of intensity resonance can be observed. In the band where two-mode generation switches to one-mode (the points E and D in Fig. 22c) the level of natural intensity fluctuations sharply grows. A drop (Fig. 22c) in the dependence of $\langle \delta I_1^2 (\omega = 10\ \text{kHz})\rangle^{1/2}$ on u appears at the center of one--mode generation.

The structure observed in natural intensity fluctuations is formed due to essential reduction of the level of natural intensity fluctuations in a one-mode regime in comparison with that of two-mode generation for strong intermode coupling. It was shown in Sec. 1 that at the boundaries of two-mode generation natural intensity fluctuations sharply grow. This explains the sharp increase of natural intensity fluctuations in the points E and D in Fig. 22b.

It is shown in [30] that, in the case of a symmetric mode position with respect to the center of absorption line curve, the third resonance is observed in the dependence of $\langle I_1 \rangle$ on u for different mode quality factors. In the same situation the third resonance directed up can be seen in the dependence of $\langle \delta I_1^2 (\omega = 10\ \text{kHz})\rangle^{1/2}$ on u (see Fig. 23). A growth in intensity resonance is accompanied by an increase in the resonance of natural intensity fluctuations. A growth in spectral density of natural intensity fluctuations at low frequencies and coincidence of one of the modes with the center of the absorption line (or at $u = \pm\omega_{12}/2$) is accompanied by a reduction of the spectrum width of the natural intensity fluctuations. There are no qualitative changes in the spectrum profile for this case, i.e., CH_4 does not affect the spectrum profile of natural intensity fluctuations.

Thus, the experimental and theoretical studies presented here argue that an intraresonator methane cell enhances the influence of natural intensity fluctuation sources on natural intensity fluctuations near tunings of one of the modes to the center of the absorption line. This leads to formation of non-linear noise resonances in natural intensity fluctuations. These resonances, unknown until now, can be employed in non-linear super high resolution spectroscopy.

3.6. Effect of CH_4 on Natural Fluctuations of Intermode Beating Frequency

Natural fluctuations of intermode beating frequency are investigated in a He–Ne / CH_4 laser (λ = 3.39 μm) which generates two linearly and orthogonally polarized modes (interaction between the modes can be smoothly changed).

The experimental set-up is similar to that described in Sec. 3 and allows one to investigate the dependence of the level of spectral density of natural intermode beating frequency fluctuations at the fixed frequency ω' $\langle \delta\psi^2 (\omega = \omega') \rangle^{1/2}$ on mode position in a gain line circuit.

The study of $\langle \delta\psi^2 (\omega = \omega') \rangle^{1/2}$ is performed for a low-power He–Ne / CH_4 laser (its natural noises are sufficiently high and substantially exceed receiver noises).

Fig. 24 presents experimental dependences of $\langle \delta\psi^2 (\omega = \omega') \rangle^{1/2}$ on u (curve 1) at various values of ω' and contrast range of amplitude resonances. Fig. 24a, b, c corresponds to a small contrast range of amplitude resonances ($K \sim 5\%$) and spectral frequencies ω' = 100 kHz, 10 kHz, and 4 kHz, respectively. Fig. 24 is associated with $K = 30\%$ and ω' = 10 kHz. The dependences of the intermode beating frequency ω_{12} on u are also displayed in this figure (curve 2). Frequency resonances of width $\gamma^{(-)}$ are observed in the dependence $\omega_{12}(u)$.

The dependences $\langle \delta\psi^2 (\omega = \omega') \rangle^{1/2}$ have a resonance character. The resonance vertex coincides with the center of the methane absorption line.

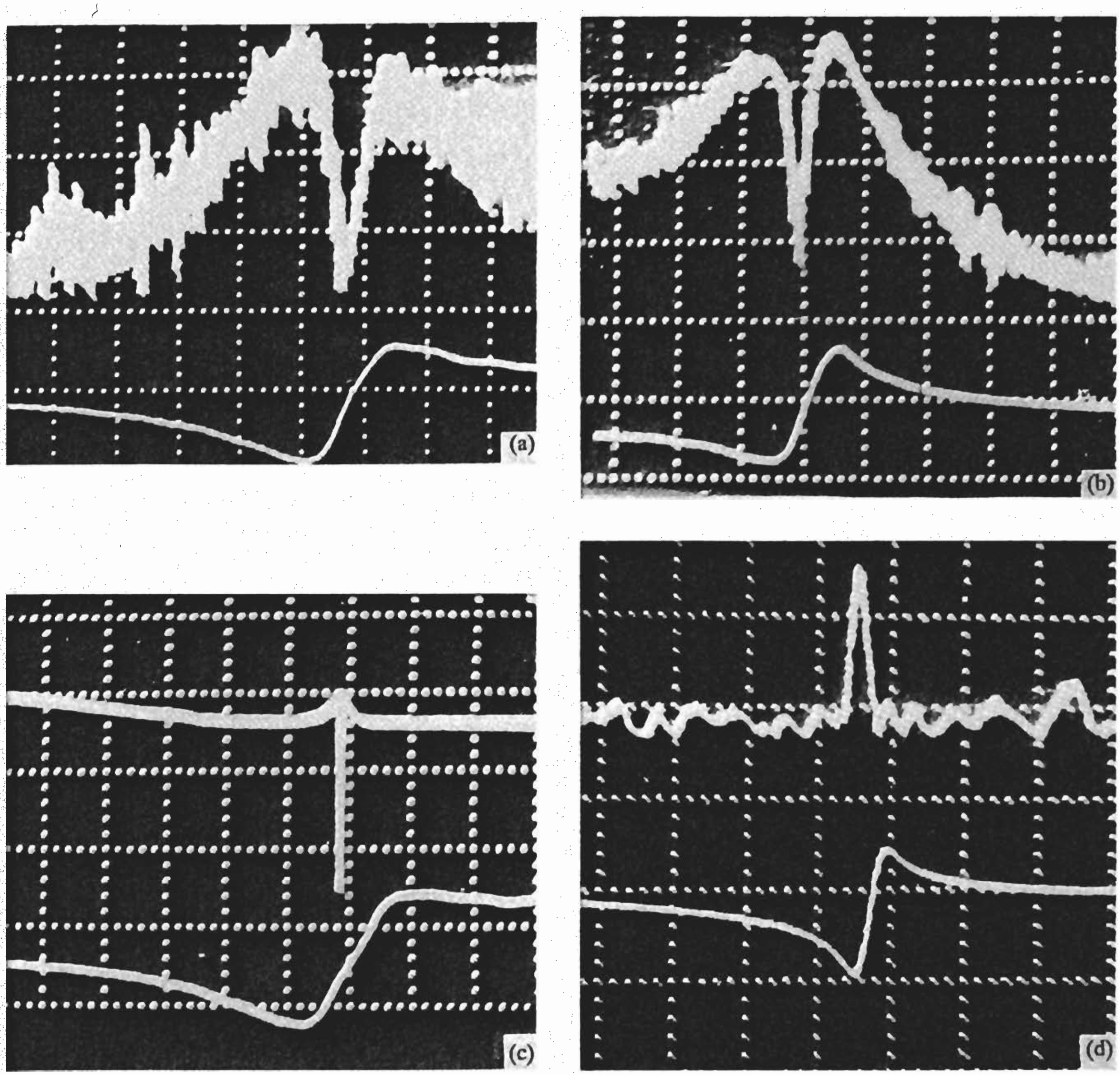

Fig. 24. *Experimental dependences of* $\langle \delta\psi^2 (\omega = \omega') \rangle^{1/2}$ *on u (curve 1) at various ω' and contrast ranges of amplitude resonances, and the dependence* $\omega_{12}(u)$*: ω' = 100 kHz, K = 5% (a), ω' = 10 kHz, K = 5% (b), ω' = 4 kHz, K = 5% (c), ω' = 10 kHz, K = 30% (d) at* $\omega_{12} = 40$ *MHz.*

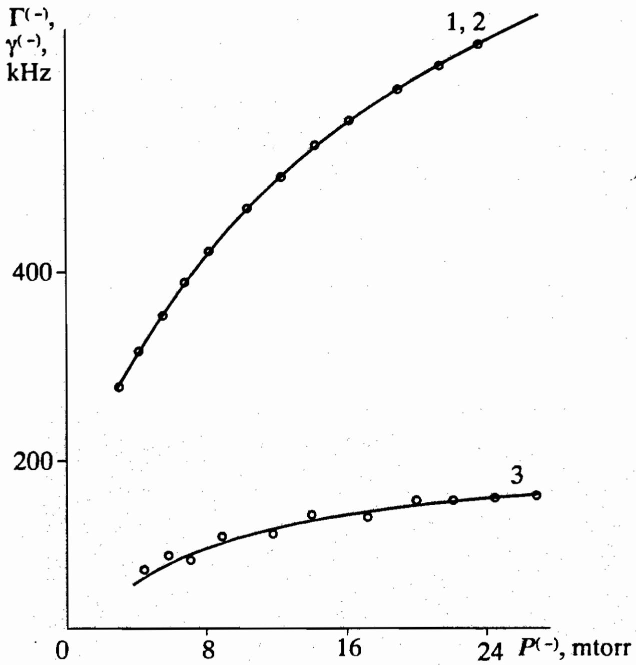

Fig. 25. *Experimental dependence of $\Gamma^{(-)}$ on $P^{(-)}$ (curves 1, 3); $\omega' = 100$ kHz (1), $\omega' = 4$ kHz (3). Dependence of $\gamma^{(-)}$ on $P^{(-)}$ (curve 2).*

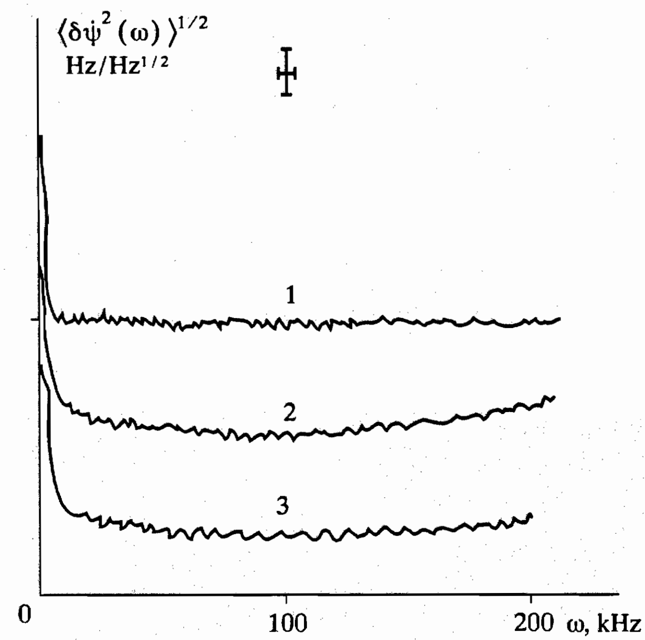

Fig. 26. *Experimental dependence $\langle \delta\dot{\psi}^2(\omega) \rangle^{1/2}$ at $\omega_{12} =$ 45 MHz: $p^{(-)} = 0$ (1), $p^{(-)} = 15$ mtorr (2), $p^{(-)} = 40$ mtorr (3).*

In accordance with experimental investigations the width ($\Gamma^{(-)}$), amplitude and profile of resonance $\langle \delta\dot{\psi}^2(\omega = \omega') \rangle^{1/2}$ essentially depend on the frequency ω' at which registration is performed and on the contrast range of amplitude resonances.

For a small contrast range of amplitude resonances, the vertex of the resonance $\langle \delta\dot{\psi}^2(\omega = \omega') \rangle^{1/2}$ is directed down. Approaching the center of the absorption line, $\langle \delta\dot{\psi}^2(\omega = \omega') \rangle^{1/2}$ at first slightly increases, but then drops dramatically to a value less than $\langle \delta\dot{\psi}^2(\omega = \omega') \rangle^{1/2}$ without CH_4.

At the frequencies $\omega' \ll \gamma^{(-)}$ the width $\Gamma^{(-)} \ll \gamma^{(-)}$ (Fig. 24c), but for $\omega' \cong \gamma^{(-)}$ the widths $\Gamma^{(-)}$ and $\gamma^{(-)}$ are equal (Fig. 24a). If the methane pressure is decreased, the width $\Gamma^{(-)}$ falls (Fig. 25). Moreover, depending on the choice of frequency ω' the tangent of the dependence $\partial\Gamma^{(-)} / \partial p^{(-)}$ changes. For $\omega' \cong \gamma^{(-)}$ the widths $\Gamma^{(-)}$ and $\gamma^{(-)}$ depend on the pressure $p^{(-)}$ in the same manner (Fig. 25, curves 1 and 2, respectively). At $\omega' \ll \gamma^{(-)}$ the width $\Gamma^{(-)}$ is much less than $\gamma^{(-)}$ in the entire interval of methane pressure 1 - 1.5 mtorr which is investigated (with a decrease in $P^{(-)}$, $\Gamma^{(-)}$ declines much more weakly than $\gamma^{(-)}$) (Fig. 25, curve 3). Therefore, a collision broadening of the resonance $\langle \delta\dot{\psi}^2(\omega = \omega') \rangle^{1/2}$ at $\omega' \ll \gamma^{(-)}$ is less than that for frequency resonance. Resonance amplitude grows as ω' falls, which is probably associated with a growth in the autostabilizing effect of methane at low frequencies.

Fig. 26 demonstrates distribution of the spectral density of natural fluctuations of intermode beating frequency with and without the presence of methane (curves 2, 3, 1, respectively) in an intraresonator absorbing cell. Curves 2 and 3 correspond to the case where one of the modes is tuned to the center of the methane absorption line. It is easy to see that for the same mode position with respect to the center of the absorption line the level $\langle \delta\dot{\psi}^2(\omega = \omega') \rangle^{1/2}$ corresponding to the presence of methane is less than the level $\langle \delta\dot{\psi}^2(\omega = \omega') \rangle^{1/2}$ without methane at any spectral frequency $\omega = \omega'$ in the band 0.4 - 200 kHz. The influence of methane on $\langle \delta\dot{\psi}^2(\omega = \omega') \rangle^{1/2}$ is stronger as its pressure grows (note that this phenomenon is essentially strong at low frequencies). This explains the increase of resonance amplitudes in $\langle \delta\dot{\psi}^2(\omega = \omega') \rangle^{1/2}$ when ω' falls.

Fig. 27 shows the dependence of $\Gamma^{(-)}$ on ω'. With a decrease in ω' the width $\Gamma^{(-)}$ reduces, and the tangent of the dependence $\frac{\partial}{\partial\omega'}\Gamma^{(-)}$ grows. At $\omega' \sim 3$ - 6 kHz the width sharply drops to a value much less than $\gamma^{(-)}$.

With a growth in the contrast range of amplitude resonances the profile of resonances in the dependence $\langle \delta\dot{\psi}^2(\omega = \omega') \rangle^{1/2}$ changes. At large K the resonance in $\langle \delta\dot{\psi}^2(\omega = \omega') \rangle^{1/2}$ is directed up (Fig. 24d). Its width does not depend on ω' any more and at any observation frequencies ω' coincides with $\gamma^{(-)}$. As K further

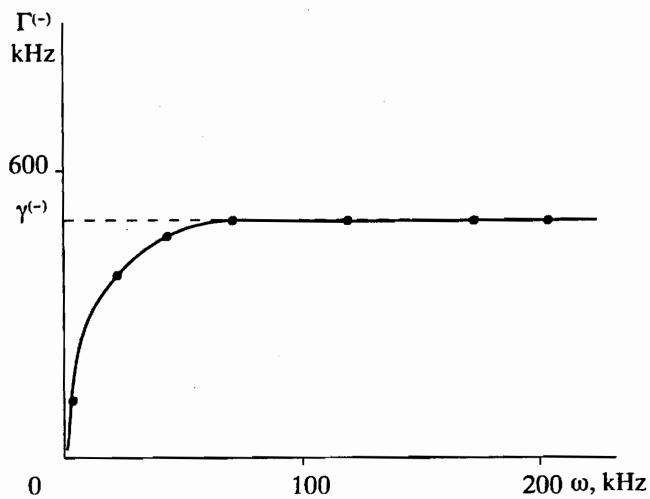

Fig. 27. *Experimental dependence of $\Gamma^{(-)}$ on ω' at $P^{(-)}$ = 15 mtorr, ω_{12} = 45 MHz, δ = 75°.*

increases, so does the resonance amplitude in the dependence $\langle \delta\psi^2(\omega = \omega') \rangle^{1/2}$.

Thus, CH_4 affects natural fluctuations of intermode beating frequency due to the following reasons: (i) a change in mode intensity (intensity resonances) (in this case resonances in $\langle \delta\psi^2(\omega') \rangle^{1/2}$ are directed up and are of width ~ $\gamma^{(-)}$), (ii) autostabilization effect (in this case resonances with width $(\gamma^{(-)} - \gamma_0^{(-)}) \ll \gamma^{(-)}$ can be observed in the dependence $\langle \delta\psi^2(\omega') \rangle^{1/2}(u)$).

One can qualitatively understand the formation of narrow structures in $\langle \delta\psi^2(\omega') \rangle^{1/2}(u)$ with widths ~ $|\gamma^{(-)} - \gamma_0^{(-)}|$ as follows.

Fluctuation variation of the population of operating level in methane $\delta\rho_{aa}(t)$ obeys the law

$$\delta\rho_{aa}(t) = \delta\rho_{aa}(t = 0)\, l^{-\gamma_0^{(-)} t}.$$

In turn, population fluctuation results in fluctuations of medium polarization $\delta\rho_{ab}(t)$, i.e., polarization fluctuations can be treated as an impressed disturbing force for polarization fluctuations.

The process of medium polarization relaxation caused by polarization variation is described by the equation [35]

$$\left(\frac{\partial}{\partial t} + v\frac{\partial}{z} - i\omega_0 + \gamma^{(-)}\right)\delta\rho_{ab} = f(t),$$

where $f(t)$ is the impressed disturbing force

$$f(t) = (\dot{\varepsilon}\mathbf{d})\,\delta\rho_{aa}(t = 0)\, l^{-\gamma_0^{(-)} t}\varphi(\mathbf{v});$$

$$\dot{\varepsilon} = \mathbf{l}E(t)\left(\frac{l^{i\omega_1 t} + l^{-i\omega_1 t}}{2}\right)\left(\frac{l^{ikz} - l^{-ikz}}{2}\right),$$

where d is the dipole moment of methane, and $\varphi(\mathbf{v})$ is the Maxwell distribution function.

The solution of this equation for $\delta\rho_{ab}$ can be expressed as

$$\delta\rho_{ab}(t) = A_1 l^{i\omega_1 t + ik_1 z} + A_2 l^{i\omega_1 t - ik_1 z} + A_3 l^{-i\omega_1 t + ik_1 z} + A_4 l^{-i\omega_1 t - ik_1 z},$$

where

$$A_1 = \frac{l^{-i(k_1 v - \omega_0)t - \gamma^{(-)} t}}{i(k_1 v - \omega_0 + \omega_1) + (\gamma^{(-)} - \gamma_0^{(-)})} \text{ etc.}$$

The solution of the equation for $\delta\rho_{ab}(t)$ shows that the value of medium polarization fluctuation depends on coincidence of the damping decrement for impressed disturbing force $\gamma_0^{(-)}$ with the damping decrement for medium polarization ($\gamma^{(-)}$) and detuning of a mode from the center of the absorption line. The influence of polarization fluctuations of the absorbing medium on variation of polarization grows dramatically, if relaxation times for population and polarization become equal. In this case the level of medium polarization variation caused by population fluctuations is maximum, if the generation frequency coincides with the center of the absorption line. For this situation polarization variation caused by relaxation processes as a function of ω_{10} can be written

$$\left[\omega_{10}^2 + (\gamma_0^{(-)} - \gamma^{(-)})^2\right]^{-1}.$$

Since in methane $\gamma_0^{(-)} \cong \gamma^{(-)}$, the width $(\gamma^{(-)} - \gamma_0^{(-)})$ for resonance observed is less than the homogeneous width of the absorption line.

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